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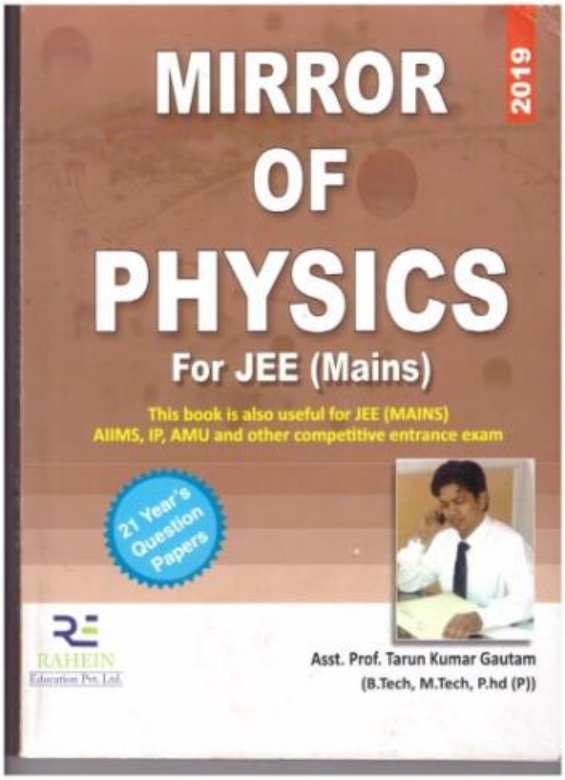
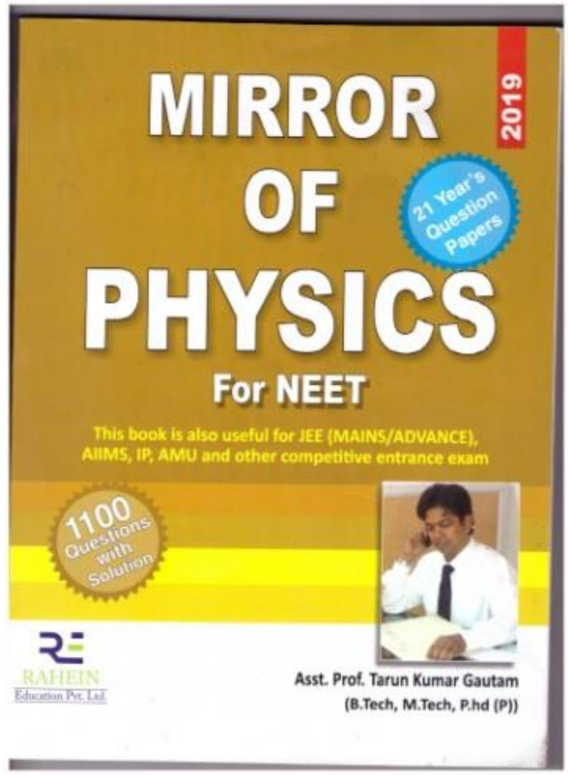
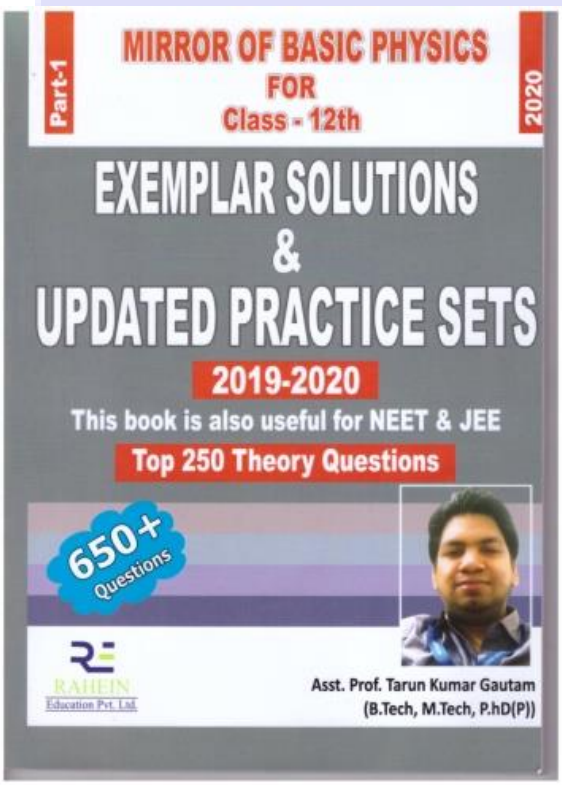
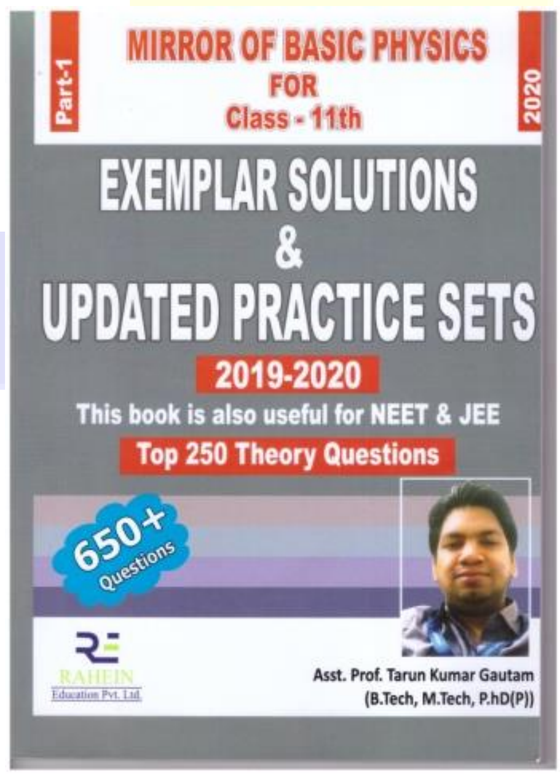






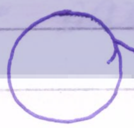


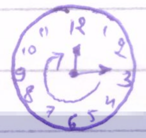
Chapter-14

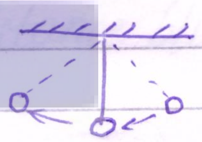
[Oscillation]

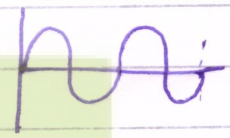
① Non-Repetition Motion
(a) Rectilinear motion (b) Projectile motion

② Repetition Motion
(a) Rotatary motion (b) Periodical motion
ex- A body move in a circle ex- wave motion



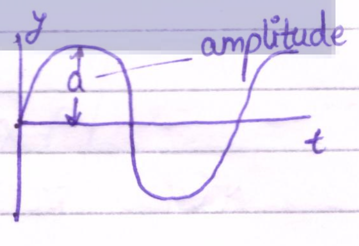




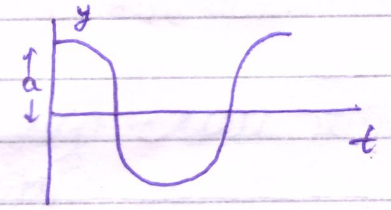


③ Equation shows periodal motion

$$y = a\sin(\omega t)$$



$$y = a\cos(\omega t)$$



Periodic Motion

Periodic Motion of a body is that motion which is repeaded identically after a fixed interval of time after which motion is repeated called "period of Motion".

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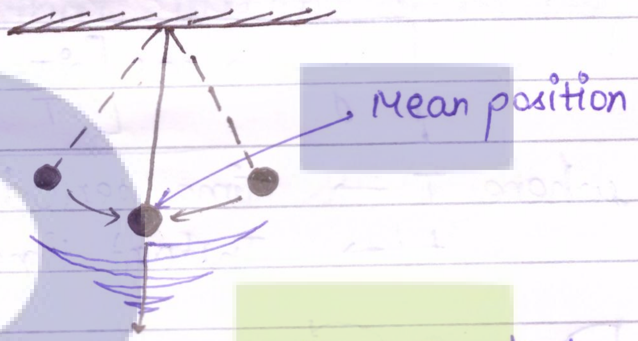
Ex–(i) Revolution of Earth around the sun. [T = 1 Yr]
(ii) Revolution of Moon around the earth [T = 27.3 days]

Oscillatory motion / Vibratory motion

It is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position or equilibrium position) in definite interval of time.



Ex – motion of pendulum of wall clock.
Ex – motion of loaded spring when a load attached to spring.
Ex – Motion of liquid in U-tube.

Harmonic Oscillation

It is that oscillation which can expressed in terms of single harmonic function (i.e, sine function or cosine function).

Single harmonic function Ex - sine function Ex - cosine function

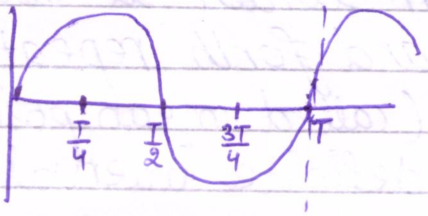
$y = a \sin\theta \longrightarrow \omega$ = angular velocity $= 2\pi\nu$

$y = a \sin(\omega t) \longrightarrow \omega = \frac{2\pi}{T}$

T ⟶ Time period

$$\boxed{y = a \sin\left[\frac{2\pi}{T} \cdot t\right]}$$

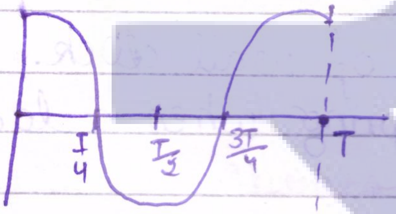
where $T \rightarrow$ Time period
$t \rightarrow$ Instant time



$t = \frac{T}{4}, t = \frac{T}{2}$ ......

$$\boxed{y = a \cos\left[\frac{2\pi}{T} \cdot t\right]}$$

where $T \rightarrow$ Time period
$t \rightarrow$ Instant time



$t = \frac{T}{4}, \; t = \frac{T}{2}, \; t = \frac{3T}{4}$

Non Harmonic Oscillation

It is that Oscillation that which cannot expressed in term of Single harmonic function.

$$\boxed{y = a \sin(\omega t) + b \cos(\omega t)}$$

Periodic Function

Periodic Function are those which are used to represent periodic motion

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$$\boxed{f(t) = f(t+T) = f(t+2T) = f(t)}$$

Ex— $\sin\theta = \sin(\theta+\pi)$ — function
$\sin(\theta+2\pi)$

Phase

It is a physical quantity which completely express the position and direction of motion of particle at that instant with respect to mean position.

Ex- $y_1 = a\sin\theta = a\sin\left(\frac{2\pi}{T}\cdot t\right)$

$y_2 = a\sin\left(\frac{2\pi}{T}\cdot t \pm \phi\right)$

Path difference → $\Delta y = y_2 - y_1$

Phase Difference

Between two vibrating particles tells the lack of harmony in vibrating state.

$$y_1 = a\sin\left(\frac{2\pi}{T}\cdot t + \phi_1\right)$$

$$y_2 = a\sin\left(\frac{2\pi}{T}\cdot t + \phi_2\right)$$

$$\Delta\phi = \phi_2 - \phi_1$$

Charactersketch of Simple Harmonic Motion

① displacement : $y = a\sin(\omega t) \Rightarrow \left[\delta - \omega t = \frac{y}{a}\right]$

$y = a\cos(\omega t)$

② Velocity : $v = \frac{dy}{dt} = \frac{d(a\sin\omega t)}{dt}$

$$V = a\frac{d(\sin\omega t)}{dt} = a \times \cos\omega t \cdot \omega$$

$$V = a\omega\cos(\omega t)$$

$$V = a\cdot\omega\sqrt{1^2 - \sin^2\omega t}$$

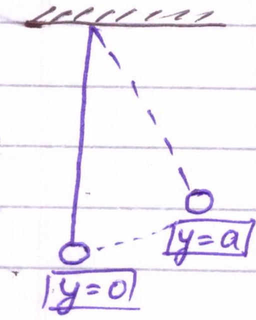
$$V = a\cdot\omega\sqrt{1 - \frac{y^2}{a^2}}$$

$$V = a\cdot\omega\sqrt{\frac{a^2 - y^2}{a^2}} \Rightarrow \frac{\cancel{a}\cdot\omega}{\cancel{a}}\sqrt{a^2 - y^2}$$

$$\boxed{V = \omega\sqrt{a^2 - y^2}}$$

at mean position $\rightarrow$ $y = 0$, $\boxed{V = \omega a}$

at extreme position $\rightarrow$ $y = a$, $\boxed{V = 0}$



③ Accelaration : $A = \frac{dv}{dt} = \frac{d(a\omega\cos\omega t)}{dt}$

$$A = a\omega\frac{d(\cos\omega t)}{dt}$$

$$A = -a\omega\sin\omega t\,(\omega)$$

$$A = -\omega^2\cdot a\cdot\sin\omega t$$

$$A = -\omega^2[a\sin\omega t]$$
$$\boxed{A = -\omega^2 y}$$

at mean position : $y = 0$, $\boxed{A = 0}$

at extreme position : $y = a$, $\boxed{A = -\omega^2 a}$

④ Time period : $A = \omega^2 y$

$$\omega = \sqrt{\frac{A}{y}}$$

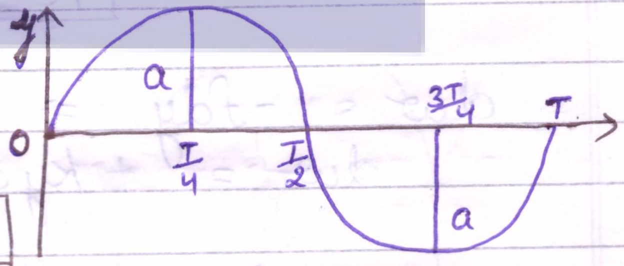
$$\omega = 2\pi\nu \Rightarrow \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{y}{A}}}$$

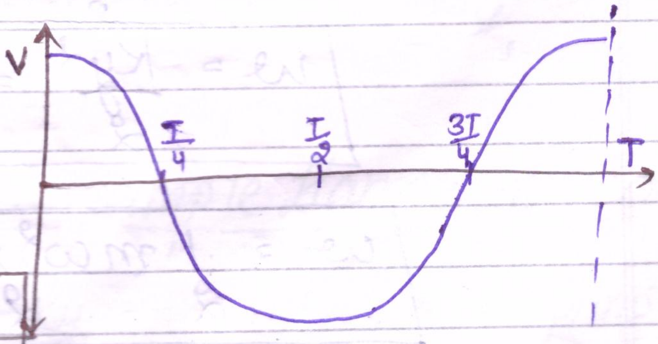
Graphical Representation

displacement : y

t	T/4	T/2	3T/4	T
y	a	0	a	0



velocity : V



t	T/4	T/2	3T/4	T
V	0	$-a\omega$	0	$+a\omega$

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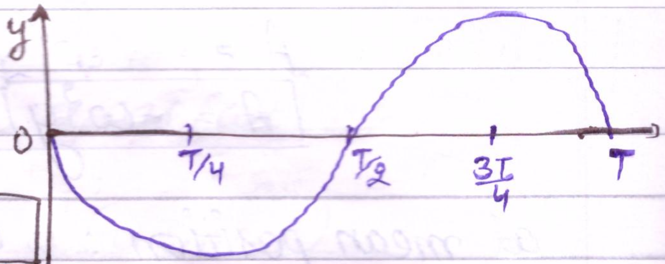
Accelaration : A



t	T/4	T/2	3T/4	T
A	$-a\omega^2$	0	$a\omega^2$	0

Total Energy in Simple Harmonic Motion

① P.E $\Rightarrow$ $y = a\sin\omega t$

$$V = \frac{dy}{dt} = a\omega\cos\omega t$$

$$A = \frac{dv}{dt} = -a\omega^2\sin\omega t \Rightarrow -\omega^2 y$$

$$f = ma$$

$$f = -m\omega^2 y \Rightarrow -Ky$$ → spring constant

$$\boxed{K = m\omega^2}$$

$$dw = -fdy \Rightarrow -(-ky)dy$$

$$dw = +Kydy$$

$$\int dw = \int_0^y Kydy$$

$$\boxed{w = \frac{Ky^2}{2} = \frac{m\omega^2 y^2}{2}}$$

$$\boxed{w = \frac{1}{2}\frac{m\omega^2 \times a^2\sin^2\omega t}{2}}$$

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(2) $K.E = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}m(a\omega\cos\omega t)^2$

$K.E = \frac{1}{2}m\omega^2 \times a^2 \times \cos^2\omega t$

$KE = \frac{1}{2}Ka^2 \times \cos^2\omega t$

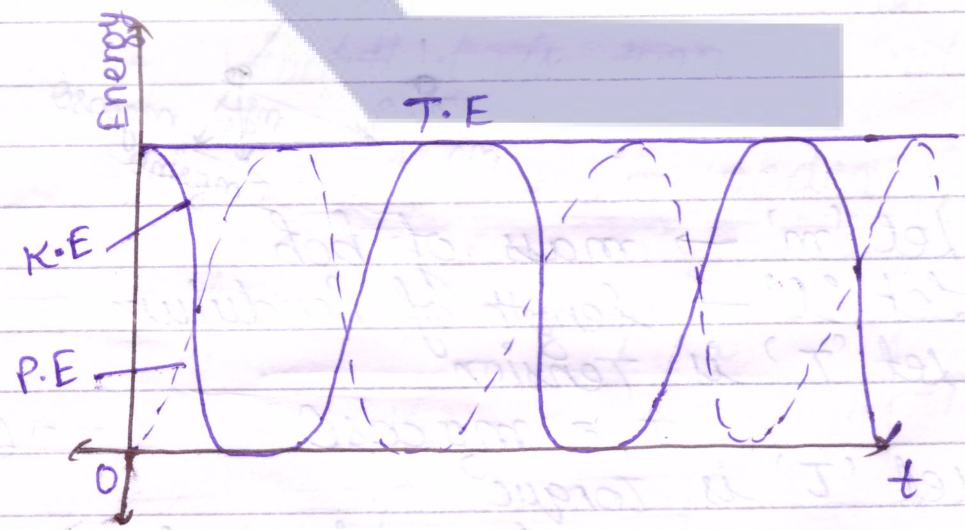
$K.E = \frac{1}{2}Ka^2 \times (1 - \sin\omega t)$

$$\boxed{K.E = \frac{1}{2}Ka^2\left[1 - \frac{y^2}{a^2}\right] \Rightarrow \frac{1}{2}m\omega^2(a^2 - y^2)}$$

$T.E = P.E + K.E$

$T.E = \frac{1}{2}Ky^2 + \frac{1}{2}K(a^2 - y^2)$

$$\boxed{T.E = \frac{1}{2}Ka^2 = \frac{1}{2}m\omega^2a^2}$$



Time period in Simple Harmonic Motion

$f = -Ky$ ---- (1) $\left[\begin{array}{l} K \rightarrow \text{spring constant} \\ y \rightarrow \text{displacement} \end{array}\right]$

$F = m \times A$

$$F = m\omega^2 y \text{ ------ } (2)$$

$[A = \text{Acceldration} = \omega^2 y]$

Combing eqⁿ (1) & (2)

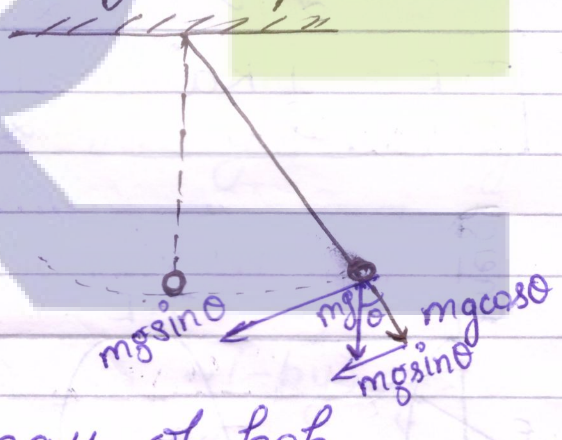
$$K = \omega^2 m$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{K}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\text{Inertial factor}}{\text{spring constant}}}}$$

Time Period of Simple Pendulum



Let 'm' → mass of bob
Let 'l' → length of Pendulum
Let 'T' is Tension

$$T = mg\cos\theta \text{ ------ } (1)$$

Let 'τ' is Torque

$$\tau = (-mg\sin\theta) \times l$$

$[\theta = \text{small}]$ $\sin\theta \simeq \theta$

So, $\boxed{\tau = -mgl\theta}$

$\rightarrow$ $\boxed{\tau = K \times \theta}$, $\boxed{K = mgl}$

M $\rightarrow$ Inertial factor
$\rightarrow$ moment of Inertia about the point of suspension

$\boxed{M = m \times l^2}$

$\boxed{K = mgl}$ $\rightarrow$ $\boxed{T = 2\pi\sqrt{\frac{M}{K}}}$

$$T = 2\pi\sqrt{\frac{\cancel{m}l^{\cancel{2}}}{\cancel{m}g\cancel{l}}}$$

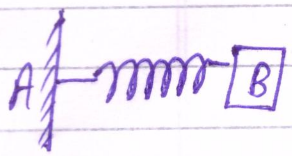
$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Second Pendulum

It is that simple pendulum whose time period of vibration is two second.

Oscillation in Loaded Spring

1) Vibration of horizontal Spring :



$$f = -Ky$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}} = \sqrt{\frac{m}{K}}}$$

$$\boxed{\nu = \text{frequency} = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

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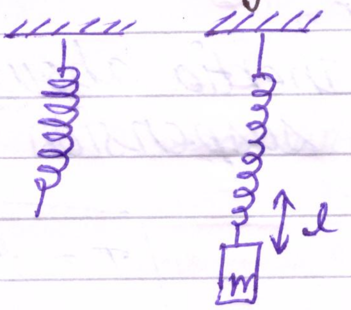
2) Vibration of Vertical Spring:



Let f_1 is restoring force set up into spring then,

$$f_1 = -k \times l$$
$$f_1 = mg$$

$$mg = kl$$

$$\boxed{k = \frac{mg}{l}}$$

$$f_2 = -k(l+y)$$

$$F = F_2 - f_1$$
$$F = -k(l+y) - (-kl)$$

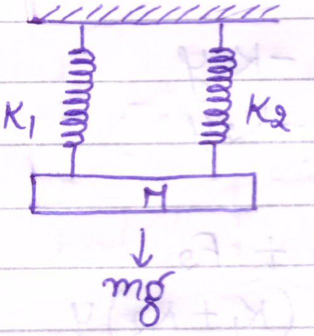
$$\boxed{F = -ky}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{\frac{mg}{l}}} = 2\pi\sqrt{\frac{l}{g}}}$$

$$\boxed{\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

★ Oscillation of Loaded Spring Combination

C-I :



$$F_1 = -K_1 y$$
$$F_2 = -K_2 y$$

$$F = F_1 + F_2$$

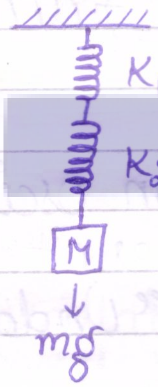
$$F = -(K_1 + K_2)\,y \quad \text{-------- (1)}$$
$$F = -Ky \quad \text{-------------- (2)}$$

Comparing eq^n (1) & (2)

$$K = K_1 + K_2$$

$$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow 2\pi\sqrt{\frac{m}{K_1 + K_2}}$$

C-II :



$$F = -K_1 y_1$$
$$y_1 = \frac{-F}{K_1}$$

$$F = -K_2 y_2$$
$$y_2 = \frac{-F}{K_2}$$

$$y = y_1 + y_2$$

$$y = \frac{-F}{K_1} - \frac{F}{K_2}$$

$$y = -F\left[\frac{K_1 + K_2}{K_1 K_2}\right]$$

$$F = -y\left[\frac{K_1 K_2}{K_1 + K_2}\right], \quad K = \frac{K_1 K_2}{K_1 + K_2}$$

$$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow 2\pi\sqrt{\frac{m(K_1 + K_2)}{K_1 K_2}}$$

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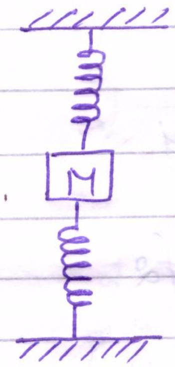
C–III :



$F_1 = -K_1 y$

$F_2 = -K_2 y$

$F = F_1 + F_2$

$F = -(K_1 + K_2)y$ ------- ①

$F = -Ky$ ------- ②

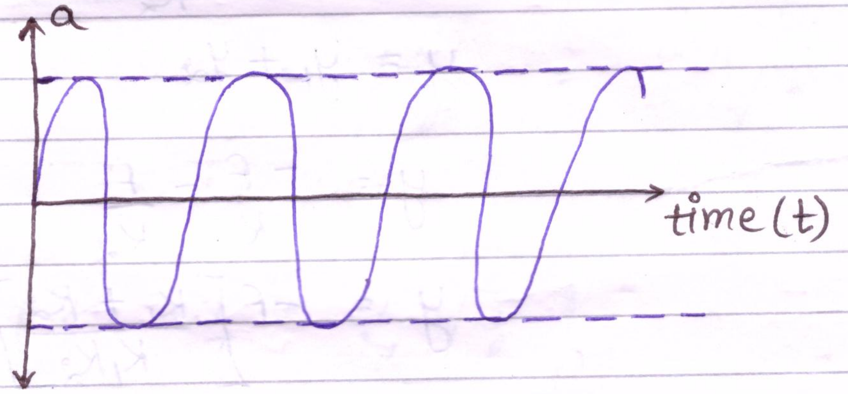
$K = K_1 + K_2$

$T = 2\pi\sqrt{\frac{m}{K}}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K_1 + K_2}}}$$

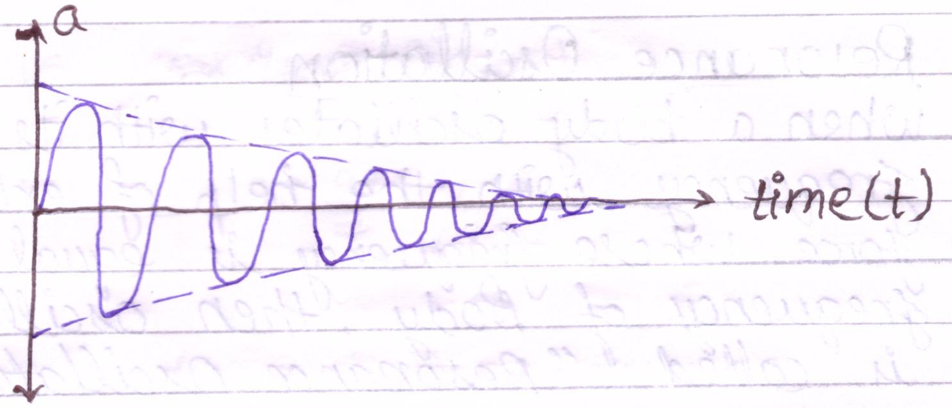
Undamped Simple Harmonic Motion

When a Simple Harmonic motion oscillates with constant amplitude which doesn't change with time, its oscillation called "Undamped SHM"



Damped Simple Harmonic Motion

When a simple Harmonic system oscillated with decreasing amplitude with time, its oscillations are called "Damped oscillation"



Free Oscillation

A system of oscillating said to be executing free oscillation if it vibrates with it own natural frequency without help of any external periodic force.

$$\boxed{V_0 = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

Ex - Oscillation of prongs of a tuning fork
Ex - Oscillation of Bob in pendilum.
Ex - Oscillation of string of Sitar.

Forced Oscillation

When a Body oscillates with help an external periodic force with frequency different form a natural frequency of Body its oscillation are called forced Oscillation

Ex - Sound Board of all strings musical instrument.

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Resonance Oscillation

when a body oscillates with its own natural frequency with the help of external periodic force whose frequency is equal to Natural frequency of Body, then oscillation of Body is called "Resonance Oscillation".

Ex- Sonometer experiment
Ex- In resonance apparatus.

Ques A Body Oscillates with SHM

$$x = 5\cos\left[2\pi t + \frac{\pi}{4}\right], \quad t = 1.5 \text{ sec}$$

(a) displacement
(b) Speed
(c) Accelaration

Ans $$x = 5\cos\left[2\pi t + \frac{\pi}{4}\right]$$

(a) $$x = 5\cos\left[2\pi(1.5) + \frac{\pi}{4}\right]$$

$$= 5\cos\left[3\pi + \frac{\pi}{4}\right]$$

$$= -5\cos\frac{\pi}{4} \Rightarrow -5 \times \frac{1}{\sqrt{2}} \Rightarrow \frac{-5}{\sqrt{2}} \text{ Ans}$$

(b) $$v = \frac{dx}{dt}$$

$$v = -5\sin\left[2\pi t + \frac{\pi}{4}\right] \times 2\pi$$

$$v = -5\sin\left[2\pi(1.5) + \frac{\pi}{4}\right] \times 2\pi$$

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$$V = 5 \times 2\pi \times \sin\left(\frac{\pi}{4}\right)$$

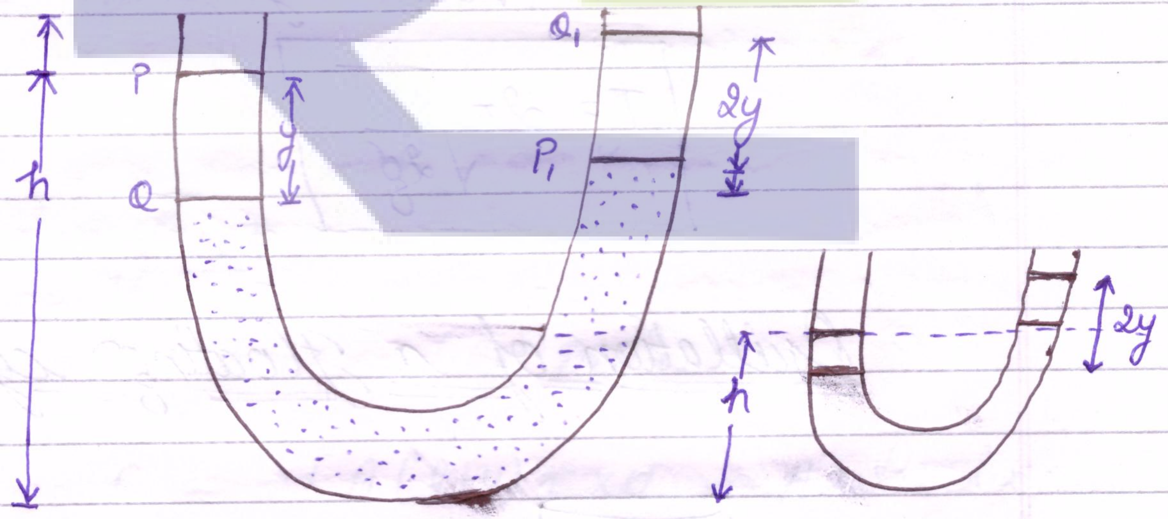
$$V = 5 \times 2 \times \frac{22}{7} \times \frac{1}{\sqrt{2}} \quad \underline{\text{Ans}}$$

(C) $$A = \frac{dv}{dt}$$

$$A = -5\cos\left[2\pi t + \frac{\pi}{4}\right] \times 2\pi \times 2\pi$$

$$A = -5 \times \frac{1}{\sqrt{2}} \times 4 \times \left(\frac{22}{7}\right)^2$$

Oscillation of Liquid in U-Tube



Let 'ρ' $\longrightarrow$ density of water

Mass of liquid in U Tube $\Rightarrow m = l \times A \times \rho$

Let : difference of levels of two limbs of U tube = $2y$

$\therefore$ [redacted] Restoring force :-

f = (weight of liquid column of height $2y$)

$$F = -(A \times 2y) \times \rho \times g$$

$$F = -(2 \times A \times \rho \times g) \times y \quad \text{------} \quad (1)$$

$$F = -ky \quad \text{------} \quad (2)$$

Comparing eq^n (1) & (2)

$$\boxed{K = 2 \times A \times \rho \times g}$$

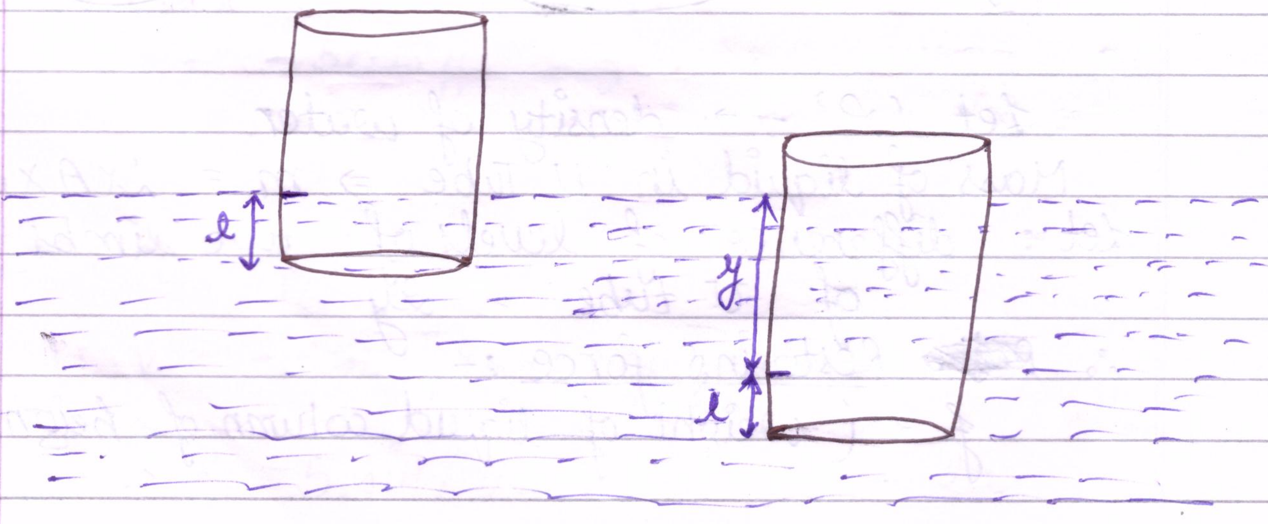
m = mass of liquid = $L \times A \times \rho$

$$T = 2\pi\sqrt{\frac{\text{inertial factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{L \times A \times \rho}{2A \times \rho \times g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{2g}}}$$

Oscillation of a Floating cylinder



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Let 'ρ' → density of material
Let 'L' → length of material
Let 'σ' → density of water

$$\boxed{m = A \times L \times \rho}$$

Upward force (f_1) acting on cylinder

$$\rightarrow \quad f_1 = (A \times L) \times \sigma \times g$$
$$f_1 = A \times \sigma \times g \times L$$

Weight of cylinder acting downward = $m \times g$

f_2 → force acting on cylinder is equal to weight of liquid displaced by length $(y + l)$ of cylinder

$$F_2 = A(l+y) \times \sigma \times g$$

Restoring force :-

$$F = -(F_2 - mg)$$

$$F = -[A(l+y)\sigma \times g - A \times \rho \times l \times g]$$

$$F = -A \times y \times \sigma \times g$$

$$F = -[A \times \sigma \times \rho] \times y \text{ ———— ----(1)}$$

$$F = -Ky \text{ -------- (2)}$$

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Comparing eq^n (1) & (2)

$$K = A\sigma g$$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{A \times L \times \rho}{A \times \sigma \times g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L \times \rho}{\sigma \times g}}}$$

—— X —— X —— X —— X —— X —

Qu What will be the Time period of second Pendulum if length is doubled.

Qu A Body weighing 10 kg is executing SHM, has velocity of 60 ms^{-1} after one second of starting from its mean position. If its time period is 6 sec.
(i) find K.E, P.E & T.E.

Ans 1 $T = 2\pi\sqrt{\frac{l}{g}} = 2$

$$T' = 2\pi\sqrt{\frac{2l}{g}} \Rightarrow \left(2\pi\sqrt{\frac{l}{g}}\right)\left(\sqrt{2}\right)$$

$$T' = 2\sqrt{2} \text{ sec}$$

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Ans 9 mass = 10 Kg
velocity = 6 m/s
$T = 6$ sec , $t = 1$ sec

$K.E = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2} \times 10 \times (6)^2 \Rightarrow \underline{180\,J}$

$v = a\omega \cos \omega t = a\omega \cos \frac{2\pi}{T}.t$

$6 = a\omega \cos \frac{2\pi}{6} \times 1 = a\omega \times \frac{1}{2}$

$\boxed{a\omega = 12}$

$T.E = \frac{1}{2}m\omega^2 a^2$

$= \frac{1}{2} \times 10 \times (12)^2 = 720\,J$

$T.E = P.E + K.E$

$P.E = T.E - K.E$

$= 720 - 180$

$= \underline{540\,J}$

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