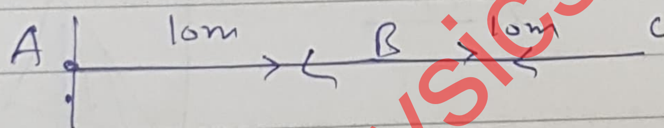
L-2

1. length → Scalar
2. Area → Scalar
3. volume → Scalar
4. distance
5. displacement

→ distance → Total Path covered

→ displacement → path b/w initial & final point

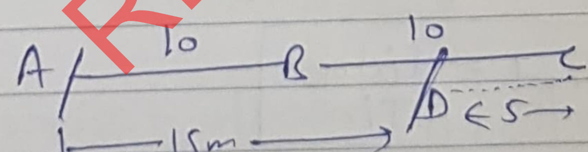
*



distance = 10 + 10 + 10 + 10 = 40m

displacement = $\cancel{10} + \cancel{10} - \cancel{10} - \cancel{10}$ = 0

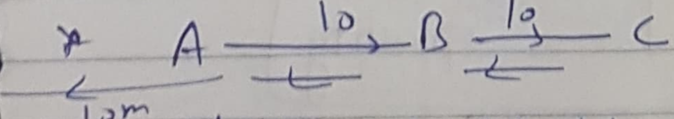
*



distance = 10 + 10 + 5 = 25m

displacement = 10 + 10 − 5 = 15

*

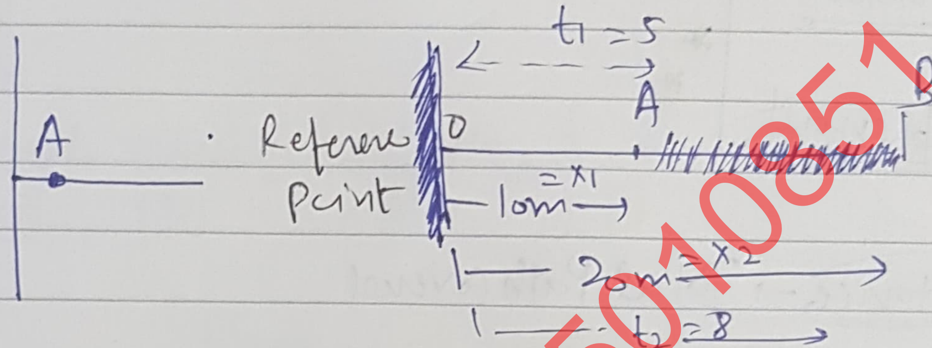


distance = 10 + 10 + 10 + 10 + 10 = 50 m

displacement = 10 + 10 − 10 − 10 − 10 = −10 m

* distance cannot be zero.
displacement may be zero, positive and Negative

→ ① Speed = $\left[\dfrac{\text{Change in distance}}{\text{Change in Time}}\right] = \dfrac{\Delta x}{\Delta t}$

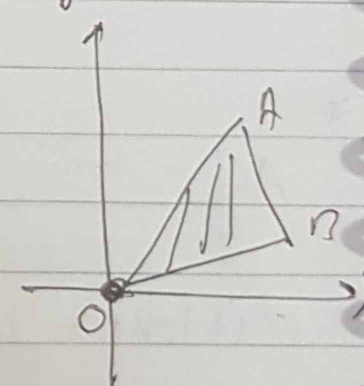


Speed = $\left[\dfrac{x_2 - x_1}{t_2 - t_1}\right] = \dfrac{\Delta x}{\Delta t}$

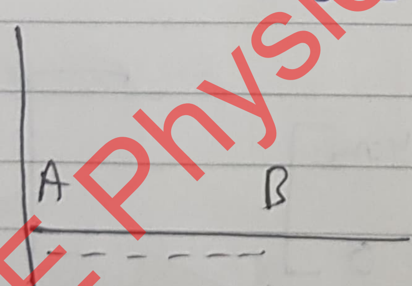
Δ → Change

ex:

$= \dfrac{20-10}{8-5} = \dfrac{10}{3}$ m/sec



* Special Case:



$x_1 = 0,\ x_2 = x$
$t_1 = 0,\ t_2 = t$

Speed = $\dfrac{x - 0}{t - 0} = \dfrac{x}{t}$

$\boxed{\text{Speed} = \dfrac{\text{distance}}{\text{Time}}}$

$\boxed{\text{Time} \rightarrow \text{Scalar}}$

Speed = $\dfrac{\text{distance}}{\text{Time}}$

$\boxed{\text{Speed} \rightarrow \text{Scalar quantity}}$

③ velocity (v) = $\dfrac{\text{change in displacement}}{\text{change in Time}}$

$$\boxed{V = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$

X → displacement

V → vector quantity

⑤ : Acceleration = $\dfrac{\text{change in velocity}}{\text{change in time}}$

$$\boxed{a = \frac{\Delta v}{\Delta t}}$$

a → vector quantity

⑥ force ⇒ $\boxed{f = \text{mass} \times \text{Acceleration}}$

m → mass → scalar quantity

→ f → vector quantity.

weight → $\boxed{w = mg}$ It always act downward

↳ vector quantity

⑦ Pressure = $\dfrac{\text{force}}{\text{Area}}$

$$\boxed{P = \frac{f}{A}}$$

Pressure is vector quantity

$\boxed{\text{L-2}}$

ch-2

units

	m.k.s	c.g.s
Mass →	kg	gram
length →	m	cm
Time →	sec	sec

→ Energy → $\boxed{\text{Joule}}$ ⟶ $\boxed{\text{erg}}$

→ force → $\boxed{\text{Newton}}$ ⟶ $\boxed{\text{dyne}}$

* m.k.s → [standard units]

ex: A Body move with 15 km/hr

Ans $15 \times \left[\frac{5}{18}\right] \rightarrow \frac{15 \times 1000}{60 \times 60} \frac{m}{sec}$

$=$

Mass → M, length → L, Time → T

		Unit	Dimension
1.	length l	→ m | cm | km	[L]
2.	Mass	→ kg | g |	[M]
3.	Time	→ sec | hr | min	[T]
4.	distance / displacement	→ m / cm / km	[L]

5. Speed = $\frac{\text{distance}}{\text{Time}}$ → $S = \frac{m}{sec}$ $\quad \left[\frac{L}{T}\right] = [LT^{-1}]$

$\boxed{S = \frac{d}{T}}$ $\quad \boxed{S = m\,sec^{-1}}$

6. Acceleration = $\frac{\Delta V}{\Delta t}$

$a = \frac{\Delta V}{\Delta t}$

$a = \frac{V}{t} = \frac{d}{t \times t} \Rightarrow \frac{m}{sec \times sec} = [m\,sec^{-2}]$ $\Rightarrow \frac{L}{T \times T} = \frac{L}{T^2}$

$= [LT^{-2}]$

7. force = $m \times a$

$= m \times \frac{v}{t}$

$= m \times \frac{d}{t \times t}$

$\rightarrow \frac{kg \times m}{sec \times sec}$

$\rightarrow \frac{kg\,m}{sec^2} = kg\,m\,sec^{-2}$

$\rightarrow \frac{M.L}{T \times T}$

$\rightarrow \frac{ML}{T^2} = [MLT^{-2}]$

3. Work = force × displacement

$= ma \times d$

$= m \times \frac{v}{t} \times d$

$= m \times \frac{d}{t \times t} \times d$

$= \frac{md^2}{T^2}$

$= \frac{md^2}{T^2}$

$= \frac{kg\,m^2}{sec^2}$

$\Rightarrow kg\,m^2\,sec^{-2}$

$= [MLT^{-2}] \times [L]$

$\Rightarrow [ML^2T^{-2}]$

All energy have same dimention & units

ex: $KE = \frac{1}{2} m v^2$ → constant

$kg \times \left(\frac{d}{t}\right)^2 = kg\left(\frac{m}{sec}\right)^2 = kg\,m^2\,sec^{-2}$

unit → $kg.\left(\frac{m}{sec}\right)^2 = kg\,\frac{m^2}{sec^2} = kg\,m^2\,sec^{-2}$

Dimention $= \frac{1}{2} m v^2$ → constant

$= M\left[\frac{L}{T}\right]^2$

$= M\frac{L^2}{T^2}$

$= [ML^2T^{-2}]$

(9) Area = $l \times b$

Unit = $m \times m = m^2$

Dimention $\Rightarrow L \times L = [L^2]$

(10) volume = $[l \times b \times h]$

Unit = $m \times m \times m = m^3$

Dimention $\Rightarrow L \times L \times L = [L^3]$

(11) Accelration due to gravity (g)

Accelration $\boxed{a = g = [LT^{-2}]}$; unit $\Rightarrow \boxed{a = g = [m\,sec^{-2}]}$

(12) Potential energy (P.E).

$\rightarrow \boxed{P.E = m \times g \times h}$

unit $\rightarrow$ P.E = $kg \times [m\,sec^{-2}] \times m$

$\boxed{P.E = [kg\,m^2 sec^{-2}]}$

Dimention $\Rightarrow M[LT^{-2}] \times [L]$

$\Rightarrow [ML^2T^{-2}]$

(13) ~~Stress~~ Pressure = $\frac{\text{Force}}{\text{Area}}$

Unit: $P = \frac{[kg\ \cancel{m}\ sec^{-2}]}{m^{\cancel{2}}}$

$\rightarrow P = [kg\,m^{-1}\,sec^{-2}]$

Dimention $\rightarrow P = \left[\frac{MLT^{-2}}{L^2}\right] \Rightarrow [ML^{-1}T^{-2}]$

$$\boxed{\text{Stress} = \text{Pressure} = \frac{\text{Force}}{\text{Area}}}$$

(14) **Impulse** $\rightarrow$ force applied for short time

ex: hitting Ball By Bat

ex: Slaps

$I = \text{force} \times \text{time}$

Unit: $I = [kg\,m\,sec^{-2}] \times [sec]$

$$\boxed{I = [kg\,m\,sec^{-1}]}$$

Dimention: $I = [MLT^{-2}] \times [T]$

$$\boxed{I = [MLT^{-1}]}$$

(15) → Linear Mamentum (P)

↳ Motion contain in a body is called "Linear Mamentum"

→ P = mass × velocity

→ P = m × V

Unit:

→ $P = kg \times \left[\frac{m}{sec}\right] = [kg\,m\,sec^{-1}]$ ✓

dimention → $P = M \times \left[\frac{L}{T}\right]$

$$\boxed{P = M[LT^{-1}]}$$

(16) $Power = \frac{work}{Time}$, $\boxed{P = \frac{W}{T}}$

→ $P = \frac{force \times distance}{(Time)} = \frac{(kg\,m\,sec^{-2}) \times (m)}{(sec)}$

→ $P = (kg\,m^2 sec^{-2} sec^{-1})$

→ $\boxed{P = (kg\,m^2 sec^{-3})}$

Dimention.

$$\boxed{P = [ML^2T^{-3}]}$$

$P = \frac{[MLT^{-2}] \times [L]}{[T]}$

→ $P = [ML^2T^{-2}T^{-1}]$

→ $\boxed{P = [ML^2T^{-3}]}$ A

Note

1. Speed = $\frac{\text{change in distance}}{\text{Change in Time}} \rightarrow S = \frac{\Delta x}{\Delta t}$

2. velocity = $\frac{\text{change in displacement}}{\text{change in Time}} \rightarrow v = \frac{\Delta x}{\Delta t}$

3. Acceleration = $\frac{\text{change in velocity}}{\text{Change in Time}} \rightarrow a = \frac{\Delta v}{\Delta t}$

4. force = mass × Acceleration

5. Pressure = $\frac{\text{force}}{\text{Area}}$ = Stress

6. Impulse = force × time

7. Linear Momentum = mass × velocity

8. g = Acceleration due to gravity = a

9. KE = $\frac{1}{2}mv^2$

10. PE = $m \times g \times h$

11. work = force × distance

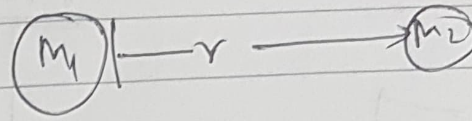
12. Power = $\frac{\text{work}}{\text{Time}}$

13. $\omega = \frac{\Delta\theta}{\Delta t}$, $\alpha = \frac{d\omega}{dt}$, $\omega = 2\pi\nu = 2\pi \times \frac{1}{T}$

Q: find the Dimention of 'G'

If

$$\boxed{f = \frac{G M_1 M_2}{r^2}}$$



A

$$\frac{F \times r^2}{M_1 M_2} = G$$

$$\frac{MLT^{-2} \times L^2}{M \times M} = G$$

$$ML^3T^{-2} \times M^{-2} = G$$

$$\boxed{M^{-1}L^3 \times T^{-2} = G}$$

$\boxed{\nu \rightarrow nu/mu}$

Q: find the Dimention of $\sin\theta$?

A: $\sin\theta \Rightarrow$

$\omega = \frac{\theta}{t}$

$\rightarrow \theta = \omega \times t$

$\rightarrow \theta = 2\pi\nu \times t$

$\theta = 2\pi \times \frac{1}{T} \times t$

$\boxed{\theta \rightarrow \text{dimentionless}}$

$\omega \rightarrow$ angular velocity

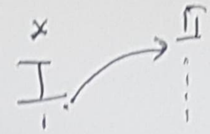
$\theta \rightarrow$ angular distance

$* \omega = 2\pi\nu$

$\nu \rightarrow$ frequency

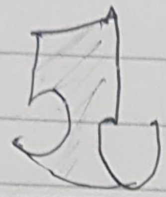
$\boxed{T = \frac{1}{\nu} = \text{Time period}}$

$\rightarrow$ $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\operatorname{cosec}\theta$, $\sec\theta$, $\rightarrow$ all are dimentionless



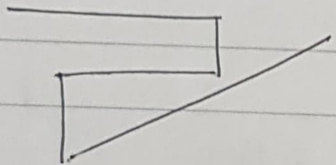
$\omega \rightarrow$ omega

$\alpha \rightarrow$ alpha

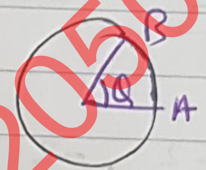


Note

Linear Motion	Rotational Motion

① 

Linear distance / Linear displacement $\rightarrow x/d$

① 

Angular distance / Angular displacement $\rightarrow \theta$

② Velocity $= \dfrac{\Delta x}{\Delta t}$

Speed $= \dfrac{\Delta x}{\Delta t}$

② angular speed $\Rightarrow \boxed{\omega = \dfrac{\Delta \theta}{\Delta t}}$

$\boxed{\omega = \dfrac{\Delta \theta}{\Delta t}}$

③ Accelaration $\rightarrow \boxed{A = \dfrac{\Delta V}{\Delta t}}$

③ angular Accelaration (α)

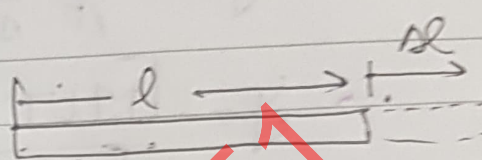
$\boxed{\alpha = \dfrac{\Delta \omega}{\Delta t}}$

Linear	Rotational
x/d →	θ
V/S →	ω
a →	α
t →	t

(17) Stress $= \frac{f}{A} = P$

$$\boxed{P = \frac{f}{A}} \rightarrow \frac{MLT^{-2}}{L^2} \Rightarrow [ML^{-1}T^{-2}]$$

(18) Strain $= \left[\frac{\text{change in length}}{\text{original length}}\right] = \frac{\Delta l}{l}$



* Unit $= \frac{m}{m} = 1$ → unitless
* Dimension → $\frac{L}{L} = 1$ → Dimensionless

(19) Young Modulus $(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{f/A}{\Delta l / l} = \left[\frac{f}{A} \times \frac{l}{\Delta l}\right]$

Unit: $\frac{\text{mass} \times \text{acceleration}}{A} \times \frac{m}{m}$; $\frac{\frac{kg \times d}{t \times t}}{d} \times \frac{m}{m}$;

$\frac{kg \times d t^{-2}}{d t^{-2}} \times 1$; $kg \times d \times d^{-1} t \times t^{-1}$ $\frac{}{t \times t}$ kg

~~Dimention~~

unit: $Y = \frac{F \times l}{A \, \Delta l} = \frac{[kg\, m\, sec^{-2}] \times [m]}{[m^2] \times [m]}$

$$\boxed{Y = [kg\, m^{-1} sec^{-2}]}$$

Note: Pressure, Stress & Young Modulus have same unit

L-3

Dimention:

$$y = \frac{f/A}{\Delta l/l}$$

$$y = \frac{f}{A} \times \frac{l}{\Delta L}$$

$$\rightarrow y = \frac{[MLT^{-2}] \times \cancel{[L]}}{[L^2] \times \cancel{[L]}}$$

$$\boxed{y = [ML^{-1}T^{-2}]}$$

↓

Application of Dimention

↓

① Check the formula
② Conversion one system into another system
③ Drive the formula

Check the formula →

① Equation of Motion

$$\boxed{V = u + at}$$

[V → final velocity
u → initial velocity
a → Accelaration
t → Time]

L.H.S → $[V] = \frac{\text{displace}}{\text{Time}} = \frac{L}{T}$

$= [LT^{-1}]$

Ans: $[u] = \left[\frac{L}{T}\right] = [LT^{-1}]$

$[at] = \left[\frac{v}{\cancel{t}} \times \cancel{t}\right] = \left[\frac{L}{T}\right] = [LT^{-1}]$

eg $\boxed{V = u + at}$

$[LT^{-1}]$ $[LT^{-1}]$ $[LT^{-1}]$

eg $5(kg) = 3(kg) + 2(kg)$

~~5kg = 5(kg x2)~~

② $\boxed{S = ut + \frac{1}{2}at^2}$

S → Distance
u → initial velocity
t → time
a → acceleration

L.H.S

→ S = distance = $[L]$

R.H.S: →

ut = $\left[\frac{\text{distance}}{\text{Time}}\right] \times \text{time}$ = $\frac{L}{\cancel{T}} \times \cancel{T} = [L]$

$\frac{1}{2}at^2$ ⇒ $at^2 = \left[\frac{V}{t}\right] \times t^2 = \left[\frac{\text{distance}}{\text{time} \times \text{time}}\right] \times \text{time}^2$

$\frac{1}{2}$ → Constant

$= \frac{L}{\cancel{T} \times \cancel{T}} \times \cancel{T^2} = [L]$

③ $\boxed{v^2 = u^2 + 2 \times a \times s}$

L.H.S: $v^2 = \left[\frac{distance}{Time}\right]^2 = \left[\frac{L}{T}\right]^2 = [L^2T^{-2}]$

R.H.S $u^2 = \left[\frac{distance}{Time}\right]^2 = \left[\frac{L}{T}\right]^2 = [L^2T^{-2}]$

$\hookrightarrow$ $2 \times a \times s = \cancel{2} \times \frac{\cancel{d}}{t \times t} \times d = \frac{d^2}{t^2} = \frac{L^2}{T^2}$

$= [L^2T^{-2}]$

Q: $x = distance$

$\boxed{x = f \times t}$

Check This correct & not ?

A

LHS: $x = distance = [L]$

RHS $F \times t$

$= MLT^{-2} \times T = MLT^{-1}$

$\boxed{LHS \neq RHS}$

This formula is wrong.

~~Imp~~ Q => $\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$ Check whether this formula is correct or not?

A. LHS. Time = [T] ✓

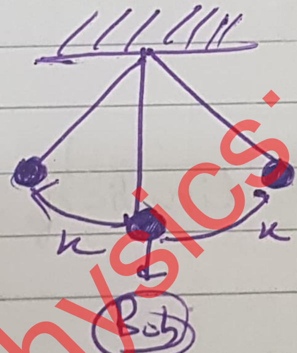
RHS. $2\pi\sqrt{\frac{l}{g}}$

constant

[l = length
g = a]

$\Rightarrow \sqrt{\frac{l}{g}} = \sqrt{\frac{L}{LT^{-2}}} = \sqrt{T^2} = [T]$ ✓

Pendulum



Q: $f = ma + (y)$ This formula is correct

f → Force
m → mass
a → Accln.

A so, dimension is equal to force ⇒ $[MLT^{-2}]$

Q: $\frac{f}{m} = y + x \times t$

find the Dimention of (x) & (y) ?

A: $y = \frac{f}{m} = \frac{\cancel{M}LT^{-2}}{\cancel{M}} = [LT^{-2}]$

$x \times t = \frac{f}{m} = \frac{\cancel{M}LT^{-2}}{\cancel{M}} \rightarrow x \times t = LT^{-2}$

$x = \frac{LT^{-2}}{T}$ $[LT^{-3}]$

Q $P \times f = y + v \times K$

P → Pressure, f → force

v → velocity, then find the dimention of (y) & (K)

Ans

$\rightarrow P \times F = y$

$\rightarrow \frac{M\cancel{L}T^{-2}}{L^{\cancel{2}}} \times M\cancel{L}T^{-2} = y$

$\rightarrow \boxed{[M^2T^{-4}] = y}$

$\rightarrow V \times K = M^2T^{-4}$

$\rightarrow \frac{L}{T} \times k = M^2T^{-4}$

$\rightarrow \boxed{k = M^2T^{-4} \times \frac{T}{L} = [M^2L T^{-5}]}$

Q Chek the formle

$$\boxed{\frac{1}{2}mv^2 = m \times g \times h}$$

A

$\frac{1}{2}$ = constant.

$$mv^2 = m \times g \times h$$

$$\left[\frac{L}{T}\right]^2 = LT^{-2} \times L$$

$$L^2T^{-2} = L^2T^{-2}$$

Hence, LHS = RHS

$g = a$

$\frac{d}{t \times t} = LT^{-2}$

LHS: $\frac{1}{2}mv^2 = \frac{1}{2}mv^2 \Rightarrow M \times \left[\frac{L}{T}\right]^2 \Rightarrow [M \times L^2T^{-2}]$

RHS: $m \times g \times h = M \times [LT^{-2}] \times [L] = [ML^2T^{-2}]$

L.H.S = R.H.S

* [Conversion one system into another system]

eg → $5\,kg = ?\,gram$

$n_1 \quad u_1 \quad n_2 \quad u_2$

eg. $2\,m\,sec^{-1} = ?\,km\,hr^{-1}$

$n_1 \quad u_1 \quad n_2 \quad u_2$

eg. $3\,kg\,m\,sec^{-2} = ?\,g\,cm\,sec^{-2}$

$$\boxed{n_1 u_1 = n_2 u_2}$$

$$\Rightarrow \boxed{n_2 = n_1 \left[\frac{u_1}{u_2}\right]}$$

$$u_1 = M_1^a L_1^b T_1^c$$

$$u_2 = M_2^a L_2^b T_2^c$$

$$n_2 = n_1 \left[\frac{M_1^a L_1^b T_1^c}{M_2^a L_2^b T_2^c}\right]$$

$$\boxed{n_2 = n_1 \left[\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c\right]}$$

$$n_2 = n_1\left[\left(\frac{m_1}{m_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c\right]$$

Q: $\boxed{1N = ?\ dyne}$

* [Newton, dyne → unit of force]

Ans: $\underset{n_1}{1}\ \underset{u_1}{N} = \underset{n_2}{?}\ \underset{u_2}{dyne}$

Dimension → $\overset{a}{M}\ \overset{b}{L}\ \overset{c}{T^{-2}}$

$$\left[\begin{array}{l} a = 1 \\ b = 1 \\ c = -2 \end{array}\right]$$

$$\Rightarrow n_2 = n_1\left[\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c\right]$$

System - I	System - II
M_1 → kg	M_2 → gram
L_1 → m	L_2 → cm
T_1 → sec	T_2 → sec
[Newton] ↓ m.k.s	[dyne] ↓ c.g.s

$$\rightarrow n_2 = 1\left[\left(\frac{kg}{g}\right)^1\left(\frac{m}{cm}\right)^1\left(\frac{\cancel{sec}}{\cancel{sec}}\right)^{-2}\right]$$

$$\rightarrow n_2 = 1\left[\left(\frac{10^3\cancel{g}}{\cancel{g}}\right)\left(\frac{10^2\cancel{cm}}{\cancel{cm}}\right)(1)^{-2}\right] = 10^3 \times 10^2 \times \frac{1}{(1)^2}$$

mv^2

$m\left(\frac{d}{t\times t}\right)^2$

$$\boxed{n_2 = 10^5}$$

$$\boxed{1\,N = 10^5 \text{ dyne}}$$

Q $\boxed{1\,J = ?\,erg}$

A J → m.k.s unit
erg → c.g.s

[∵ Joule = energy
energy SI unit = $kg\,m^2 s^{-2}$
ML^2T^{-2}
$a = 1$, $b = 2$, $c = -2$]

$$n_2 = n_1\left[\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c\right]$$

$n_1 = 1$

ATF, $n_2 = n_1\left[\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c\right]$

$$n_2 = 1\left[\left(\frac{kg}{g}\right)^1\left(\frac{m}{cm}\right)^2\left(\frac{\cancel{sec}}{\cancel{sec}}\right)^{-2}\right]$$

$$n_2 = 1\left[\left(\frac{10^3\cancel{g}}{\cancel{g}}\right)\left(\frac{10^2\cancel{cm}}{\cancel{cm}}\right)^2(1)^{-2}\right]$$

$$n_2 = 1\left[10^3\times 10^4\times 1\right]$$

$$n_2 = 10^7 \text{ erg}$$

$$\boxed{1\,J = 10^7 \text{ erg}}$$

#

① $1J = 10^7 erg$
② $1N = 10^5 dyne$

① Joule → m.k.s, Newton → m.k.s
watt → Power → m.k.s
② erg, dyne → c.g.s

[Drivation of formulas]:

↓

Q: K.E of Body is depend upon mass of body (m) & velocity of Body (v). find the formula of Kinetic Energy?

A → $K.E \propto m^a . v^b$

→ $\boxed{KE = K . m^a . v^b}$ — ③

L.h.s : → $KE = \frac{1}{2} mv^2 = M\left[\frac{L}{T}\right]^2 = [ML^2T^{-2}]$ — ①

Constant

R.h.s → $[M]^a [LT^{-1}]^b$

→ $\boxed{M^a L^b T^{-b}}$ — ②

from eq ① & ②

→ $ML^2T^{-2} = M^a L^b T^{-b}$

$a = 1, \quad b = 2,$

Put Value in eq No ③ → $\boxed{KE = K . m^1 v^2}$

Q: Potential energy of Body is depend upon mass of Body (m), Acceleration due to gravity (g) & height (h)?
Find the formula for Potential energy?

A. ~~PE = mgh~~

$$\rightarrow PE \propto m^a . g^b . h^c$$

$$\boxed{PE = K . m^a . g^b . h^c} \quad \text{— (1)}$$

L.h.s $\rightarrow P.E = [ML^2T^{-2}]$ — (2)

RHS $\rightarrow [M]^a [LT^{-2}]^b [L]^c$

$$\rightarrow M^a . L^b T^{-2b} . L^c \Rightarrow \boxed{M^a L^{b+c} . T^{-2b}}$$

— (3) $\Rightarrow$

from eq (1) & (2)

$$\rightarrow ML^2T^{-2} = M^a L^{b+c} T^{-2b}$$

$$\rightarrow \boxed{a = 1}$$

$$c + b = 2$$

$$-2b = -2$$

$$\boxed{b = 1}$$

$$\boxed{c = 2 - b = 2 - 1 = 1}$$

putting value of a, b, c in eq (A)

$$\boxed{P.E = K . m g h}$$

Q Consider a Simple Pendulum having bob attached to a string that oscillating under action of force of gravity. Suppose the period of oscillation of Simple Pendulum depend upon its (a) length (l)

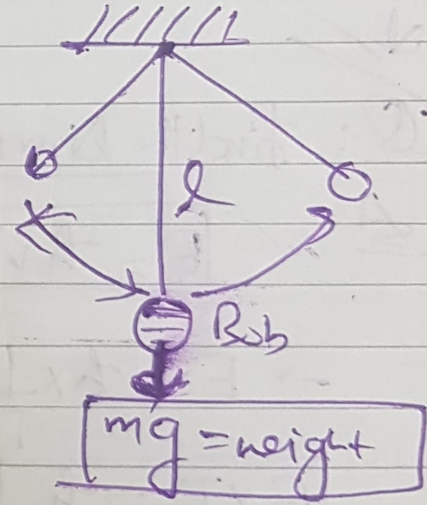
(b) mass of bob (m)

(c) Acceleration due to gravity (g)

A

$\rightarrow T \propto l^a \times m^b \times g^c$

$\rightarrow \boxed{T = K l^a . m^b . g^c}$ —— (A)

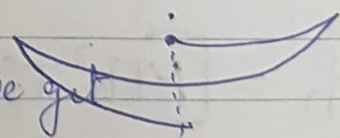


L.H.S = $[T]$ = 0 —— (1)

R.H.S = $l^a . m^b . g^c$

$= [L]^a [M]^b [LT^{-2}]^c$ —— (2)

from equation (1) and (2), we get



$\rightarrow [T] = [L]^a [M]^b [LT^{-2}]^c$

$\rightarrow [T] = L^a . M^b . L^c T^{-2c}$

$[L^0 M^0 T^1] = L^{a+c} M^b T^{-2c}$

Comparing Power.

$x^0 = 1$

$a + c = 0 \rightarrow a = -c$

$b = 0$

$-2c = 1$

$\boxed{c = -\frac{1}{2}}$

$\rightarrow \boxed{a = -\left[-\frac{1}{2}\right] = \frac{1}{2}}$

$\boxed{b = 0}$

$\boxed{a^{-1} = \frac{1}{a}}$

Putting the value of a, b, c in equation (A).

$\rightarrow \boxed{T = k\, l^{1/2} M^0 g^{-1/2}}$

$\rightarrow T = k \frac{l^{1/2}}{g^{1/2}}$

$\rightarrow T = k\left(\frac{l}{g}\right)^{1/2}$

$\rightarrow \boxed{T = k\sqrt{\frac{l}{g}}}$

Q: find the Dimension of Plank Constant (h) ?

A: $\boxed{E = h\nu}$

$[\nu \rightarrow \text{frequency}]$

$\rightarrow \nu = \left[\frac{1}{\text{Time period}}\right]$

$\rightarrow \boxed{\nu = \frac{1}{T}}$

$\rightarrow E = h \times \frac{1}{T}$

$\boxed{E \times T = h}$

$\rightarrow h = [ML^2T^{-2}] \times [T]$

$\rightarrow h = [ML^2T^{-2+1}]$

$\rightarrow \boxed{h = [ML^2T^{-1}]}$

$\rightarrow$ velocity = wavelength × frequency

$\rightarrow c = \lambda \times \nu$

$\boxed{\frac{c}{\lambda} = \nu}$

$\boxed{E = h\nu = \frac{hc}{\lambda}}$

Error

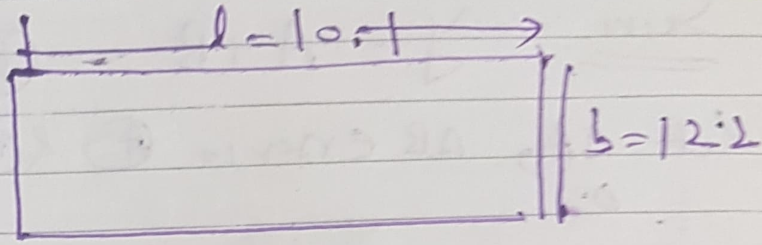
I-1 → $h_1 = 5.4$
I-2 → $h_2 = 5.5$
I-3 → $h_3 = 5.3$
I-4 → $h_4 = 5.5$
I-5 → $h_5 = 5.3$

$5.4 \xrightarrow{+0.1} 5.5$

$5.4 \xrightarrow{-0.1} 5.3$

error

* $\boxed{H = 5.4 \pm 0.1}$



$A = 10 \times 12 = 120$

$1\,m^2 \to 1\,cr$

$\boxed{A = 120\,cr}$

$A = 0.1 \times 0.2$
$= 0.02$
2 lakh ✗

ex: $A = 14 \pm 0.1$
$B = 5 \pm 0.2$

*Sum = $14 + 5 \Rightarrow 19 \pm [0.1 + 0.2]$

$\boxed{19 \pm 0.3}$ ✓

* Subtract → $(14 - 5) \Rightarrow 9 \pm [\ ?\ ]$

* Multiple – 14×5
$= 70^2 \Rightarrow 70 \pm [\ ?\ ]$

* divide = $\frac{14}{5} \Rightarrow 2.8 \pm (\ ?\ ]$
$= 2.8$

Sum: $y = A + B$ — (1)

Let ΔA, ΔB error in (A) & (B)

$\rightarrow y \pm \Delta y = (A \pm \Delta A) + (B \pm \Delta B)$

$\rightarrow y \pm \Delta y = (A + B) \pm \Delta A \pm \Delta B$

for eq (1)

$\rightarrow \cancel{y} \pm \Delta y = \cancel{A} \pm \Delta A \pm \Delta B$

$\rightarrow \pm \Delta y = \pm \Delta A \pm \Delta B$

$\rightarrow \cancel{\pm} \Delta y = \cancel{\pm}[\Delta A + \Delta B]$

$\rightarrow \boxed{\Delta y = \Delta A + \Delta B}$ Ans

Q: $A = 5 \pm 0.1$

$B = 4 \pm 0.2$

What is sum of $(A + B)$ with error?

A. $A \rightarrow 5$, $\Delta A \rightarrow 0.1$

$B \rightarrow 4$, $\Delta B = 0.2$

$\rightarrow \Delta S = \Delta A + \Delta B$

Sum $= (5 + 4) = 9$

$S = (A + B)$

$\rightarrow$ Sum with error $= S \pm \Delta S$

$= 9 \pm [0.1 + 0.2]$

→ Sum with error = $9 \pm [0.3]$

Δ

Sum:

$$\boxed{y = A + B, \quad \Delta y = \Delta A + \Delta B}$$

Note

① Sum:

$\boxed{y = A + B}$, $\boxed{\Delta y = [\Delta A + \Delta B]}$

② Subtration

$\boxed{y = A - B}$, $\boxed{\Delta y = [\Delta A + \Delta B]}$

③ Multiplication

$\boxed{y = A \times B}$, $\boxed{\frac{\Delta y}{y} = \left[\frac{\Delta A}{A} + \frac{\Delta B}{B}\right]}$

④ divide

$\boxed{y = \frac{A}{B}}$, $\boxed{\frac{\Delta y}{y} = \left[\frac{\Delta A}{A}\right] + \left[\frac{\Delta B}{B}\right]}$

⑤ $\boxed{y = \frac{A^n}{B^m}}$, $\boxed{\frac{\Delta y}{y} = \left[n\frac{\Delta A}{A} + m\frac{\Delta B}{B}\right]}$

L-5

Error Measurement

① **Sum** $\rightarrow y = A + B. \quad —(1)$

$$\underset{\Delta y}{\downarrow} \quad \underset{\Delta A}{\downarrow} \quad \underset{\Delta B}{\downarrow}$$

$\rightarrow y \pm \Delta y = (A \pm \Delta A) + (B \pm \Delta B) = (A+B) \pm \Delta A \pm B$

from eq (1)

$\rightarrow \cancel{y} \pm \Delta y = \cancel{y} \pm \Delta A \pm \Delta B$

$\rightarrow \pm y = \pm \Delta A \pm \Delta B$

$\rightarrow \cancel{\pm} y = \cancel{\pm}[\Delta A + \Delta B]$

$\rightarrow \boxed{y = \Delta A + \Delta B}$

② ~~divide~~ **Subtraction** $\rightarrow \boxed{y = A - B} \quad —(1)$

$$\underset{\Delta y}{\downarrow} \quad \underset{\Delta A}{\downarrow} \quad \underset{\Delta B}{\downarrow}$$

$\rightarrow y \pm \Delta y = (A \pm \Delta A) - (B \pm \Delta B)$

$\rightarrow y \pm \Delta y = (A - B) \pm \Delta A \mp \Delta B$

from eq (1)

$\rightarrow$

$\cancel{y} \pm \Delta y = \cancel{y} \pm \Delta A \mp \Delta B$

$\pm 2, \mp 2$

$\rightarrow \pm\Delta y = \pm\Delta A \mp \Delta B$

$\rightarrow \pm\Delta y = \pm\Delta A \pm \Delta B$

$\rightarrow \cancel{\pm}\Delta y = \cancel{\pm}[\Delta A + \Delta B]$

$\rightarrow \boxed{\Delta y = \Delta A + \Delta B}$

Note.

$\rightarrow y = A + B, \quad \Delta y = \Delta A + \Delta B$

$\rightarrow y = A - B, \quad \Delta y = \Delta A + \Delta B$

Ex: $h_1 = 15 \pm 0.2$

$h_2 = 10 \pm 0.3$

$\underline{A}$ $h_1 - h_2 = 15 - 10 = 5$

$\rightarrow 5 \pm \Delta h$

$\rightarrow 5 \pm [0.2 + 0.3]$

$\rightarrow \underline{5 \pm 0.5}$ $\underline{A}$

(3) Multiplication

$$\rightarrow \boxed{y = A \times B} \quad \text{—— (1)}$$

$$\downarrow \pm \Delta y \qquad \downarrow \pm \Delta A \qquad \downarrow \Delta B$$

$$\rightarrow (y \pm \Delta y) = (A \pm \Delta A)(B \pm \Delta B)$$

$$\rightarrow y\left[1 \pm \frac{\Delta y}{y}\right] = A\left[1 \pm \frac{\Delta A}{A}\right] B\left[1 \pm \frac{\Delta B}{B}\right]$$

$$\rightarrow y\left[1 \pm \frac{\Delta y}{y}\right] = AB\left[1 \pm \frac{\Delta A}{A}\right]\left[1 \pm \frac{\Delta B}{B}\right]$$

from eq (1)

$$\rightarrow \cancel{y}\left[1 \pm \frac{\Delta y}{y}\right] = \cancel{y}\left[1 \pm \frac{\Delta A}{A}\right]\left[1 \pm \frac{\Delta B}{B}\right]$$

$$\rightarrow 1 \pm \frac{\Delta y}{y} = \left[1 \pm \frac{\Delta A}{A}\right]\left[1 \pm \frac{\Delta B}{B}\right]$$

$$\rightarrow \cancel{1} \pm \frac{\Delta y}{y} = \cancel{1} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \underbrace{\left(\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}\right)}_{\text{Neglect}}$$

$$\rightarrow \pm \frac{\Delta y}{y} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

$$\rightarrow \boxed{\frac{\Delta y}{y} = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)}$$

④ **Divide:** $\boxed{y = \frac{A}{B}}$ — ①

$\rightarrow y \pm \Delta y = \frac{(A \pm \Delta A)}{(B \pm \Delta B)}$

$\rightarrow y \pm \Delta y = (A \pm \Delta A)(B \pm \Delta B)^{-1}$

$\rightarrow y \pm \Delta y = A\left[1 \pm \frac{\Delta A}{A}\right] \cdot B^{-1}\left[1 \pm \frac{\Delta B}{B}\right]$

$\rightarrow y\left[1 \pm \frac{\Delta y}{y}\right] = \frac{A}{B}\left[1 \pm \frac{\Delta A}{A}\right]\left[1 \pm \frac{\Delta B}{B}\right]$

$\rightarrow \cancel{y}\left[1 \pm \frac{\Delta y}{y}\right] = \cancel{y}\left[1 \pm \frac{\Delta A}{A}\right]\left[1 \pm \frac{\Delta B}{B}\right]$

$\rightarrow 1 \pm \frac{\Delta y}{y} = \left[1 \pm \frac{\Delta A}{A}\right]\left[1 \pm \frac{\Delta B}{B}\right]$

$\rightarrow \cancel{1} \pm \frac{\Delta y}{y} = \cancel{1} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \underbrace{\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}}_{\text{Neglect}}$

$\rightarrow \pm \frac{\Delta y}{y} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$

$\rightarrow \boxed{\frac{\Delta y}{y} = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)}$ A

9/4/24

* (5) $y = \frac{A^n}{B^m}$ [divide with Power]

→ $y = \frac{A^n}{B^m} = A^n \cdot B^{-m}$

Take log Both side

→ $\log(y) = \log(A^n \cdot B^{-m})$

→ $\log(y) = \log(A^n) + \log(B^{-m})$

→ $\log(y) = n\log(A) - m\log(B)$

→ Take diffchian Both side

↓

→ $\frac{dy}{y} = n \cdot \frac{dA}{A} - m\frac{dB}{B}$

→ $\boxed{\frac{\Delta y}{y} = n\left(\frac{\Delta A}{A}\right) + m\left(\frac{\Delta B}{B}\right)}$ [∵ error always positive]

Log

(1) $\log(AB) = \log(A) + \log(B)$

(2) $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

(3) $\log(A^n) = n\log A$

% error → $\boxed{\frac{\Delta y}{y} \times 100}$

Q: $y = \frac{a^2 b^3}{c^2 d^4}$ find % error in (y)

If % error in a is 1%
If " " " b is 2%
If " " " c is 3%
If " " " d is 4%

Ans: $y = a^2 b^3 . c^{-2} d^{-4}$

$$\rightarrow \frac{\Delta y}{y} \times 100 = \left[2\left(\frac{\Delta a}{a}\right) + 3\left(\frac{\Delta b}{b}\right) + 2\left(\frac{\Delta c}{c}\right) + 4\left(\frac{\Delta d}{d}\right)\right] \times 100$$

[error always positive]

$$\rightarrow \frac{\Delta y}{y} \times 100 = \left[2\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + 2\left(\frac{\Delta c}{c} \times 100\right) + 4\left(\frac{\Delta d}{d} \times 100\right)\right]$$

$$\rightarrow \frac{\Delta y}{y} \times 100 = [2 \times 1 + 3 \times 2 + 2 \times 3 + 4 \times 4]$$

$$\frac{\Delta y}{y} \times 100 = 2 + 6 + 6 + 16$$

$$= 30\% \text{ A.}$$

Q: $y = \frac{a^5 \cdot b^2}{\sqrt{c} \cdot d}$

If % a is 1%. find % error in y?
If % b is 2%
If % c is 4%
If % d is 5%

Ans

$$y = a^5 \cdot b^2 \cdot c^{1/2} \cdot d$$

$$\frac{\Delta y}{y} \times 100 = \left[5\left(\frac{\Delta a}{a}\right) + 2\left(\frac{\Delta b}{b}\right) + \frac{1}{2}\left(\frac{\Delta c}{c}\right) + \left(\frac{\Delta d}{d}\right)\right] \times 100$$

(error is always +ve)

$$\frac{\Delta y}{y} \times 100 = \left[5\times\left(\frac{\Delta a}{a}\times 100\right) + 2\left(\frac{\Delta b}{b}\times 100\right) + \frac{1}{2}\left(\frac{\Delta c}{c}\times 100\right) + \frac{\Delta d}{d}\times 100\right]$$

$$\frac{\Delta y}{y} \times 100 = \left[5\times 1 + 2\times 2 + \frac{1}{\cancel{2}}\times \overset{2}{\cancel{4}} + 1\times 5\right]$$

$$\frac{\Delta y}{y} \times 100 = [5+4+2+5]$$

$$\frac{\Delta y}{y} \times 100 = 16\%$$

A

Q: If % change in radius of circle is 2%. Then find % change in Area of circle?

A. $A = \pi r^2$

π → constant

$\rightarrow \frac{\Delta A}{A} = 2\left(\frac{\Delta r}{r}\right)$

$\rightarrow \frac{\Delta A}{A} \times 100 = 2\left(\frac{\Delta r}{r} \times 100\right)$

$= 2 \times 2$

$= 4\%$ A

Q: If % change in radius is 2% & height in 3%. find the % change in volume of cone?

A. $V = \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi$ → constant

$\rightarrow \frac{\Delta V}{V} = \left[\frac{\Delta r}{r} \times 2 + \frac{\Delta h}{h} \cancel{\times 2}\right]$

$\rightarrow \frac{\Delta V}{V} \times 100 = \left[100 \times \frac{\Delta r}{r} \times 2 + \frac{\Delta h}{h} \times 100\right]$

$\frac{\Delta V}{V} \times 100 = [2 \times 2 + 3]$

$= 4 + 3$

$= 7\%$ A

Q $y = \frac{a^2 b^3 \cdot c^{-1/2}}{\sqrt{d} \cdot e^{5/2}}$

$\Rightarrow \frac{\Delta y}{y} = a^2 b^3 c^{-1/2} d^{1/2} e^{-5/2}$

$$\frac{\Delta y}{y} \times 100 = \left[2 \times \frac{\Delta a}{a} + 3 \times \frac{\Delta b}{b} + \left(-\frac{1}{2}\right)\left(\frac{\Delta c}{c}\right) + \frac{1}{2}\left(\frac{\Delta d}{d}\right) + \left(-\frac{5}{2}\right)\left(\frac{\Delta e}{e}\right)\right] \times 100$$

$$\frac{\Delta y}{y} \times 100 = \left[2\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + \frac{1}{2}\left(\frac{\Delta c}{c} \times 100\right) + \frac{1}{2}\left(\frac{\Delta d}{d} \times 100\right) + \frac{5}{2}\left(\frac{\Delta e}{e} \times 100\right)\right]$$

Type of error

↓

1. Systematic error
2. Random error
3. gross error

* Systematic error → are those error that tends to be one direction (either or +ve or -ve)

[Cause of systematic error are known]

ex. • Instrument error
ex: Imperfection error
ex: Personal error
ex: Due to external effect/error

Random error → occur due to irregularity, due to unpreditable variable in experiment.

· Can be minimized By Repeating observation a Large No. of time.

Gross error :→ due to careless of observation

Q: What is differen Between Accuracy & Precision?

Ans

* Accuracy of a measurement is a measure of how close the measured value is to be true value of quantity

* Precision • tells us to what resolution or limit, the quantity is measured By a measuring Instrument

Note

① Absolute error

② Mean absolute error

③ Relative error / fraction error

④ Percentage error

Let $a_1 \; a_2 \; a_3 \; a_4 \cdots a_n$

Absolute error

$$\rightarrow \quad a_m = \left[\frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}\right]$$

$$\rightarrow \quad \Delta a_1 = |a_m - a_1|$$

$$\rightarrow \quad \Delta a_2 = |a_m - a_2|$$

$$\vdots$$

$$\rightarrow \quad \Delta a_n = |a_m - a_n|$$

Mean absolute error

$$\Rightarrow \quad \Delta a_m = \left[\frac{|\Delta a_1| + |\Delta a_2| \cdots + |\Delta a_n|}{n}\right]$$

1-6

(3) Relative error/ fractional error $= \dfrac{\Delta a_m}{a_m}$

(4) Percentage error $= \dfrac{\Delta a_m \times 100}{a_m}$

A

Q1 $a_1 = 2.1$
$a_2 = 1.5$ find % error?
$a_3 = 1.6$
$a_4 = 1.7$

A

$$a_m = \left[\frac{2.1 + 1.5 + 1.6 + 1.7}{4}\right] = \frac{7.0}{4} = 1.7$$

absolute error

$|\Delta a_1| = |a_m - a_1| = |1.7 - 2.1| = |-0.4| = 0.4$

$|\Delta a_2| = |a_m - a_2| = |1.7 - 1.5| = |0.2| = 0.2$

$|\Delta a_3| = |1.7 - 1.6| = 0.1$

$|\Delta a_4| = |1.7 - 1.7| = 0$

Mean absolute error

$$|\Delta g_m| = \frac{|\Delta g_1| + |\Delta g_2| + |\Delta g_3| + |\Delta g_4|}{4}$$

$$\Rightarrow \frac{0.4 + 0.2 + 0.1 + 0}{4}$$

$$= \frac{0.7}{4} = \underline{\underline{0.22}}$$

Relative error / fractional error

$$= \frac{\text{Mean absolute error}}{\text{absolute error}} \Rightarrow \frac{|\Delta g_m|}{g_m}$$

$$= \frac{0.22}{1.7}$$

Percentage error

$$\text{Percentage error} = \frac{|\Delta g_m|}{g_m} \times 100$$

$$\Rightarrow \frac{0.22}{1.7} \times 100$$

(7) Fundamental units

(1) Kilogram (2) The meter (3) second (4) Ampere

(5) Kelvin (6) Candela (unit of luminous intensity)

(7) The mole (unit of matter)

Macro units:

(1) Astronomical unit (AU): average distance b/w centre of sun to earth

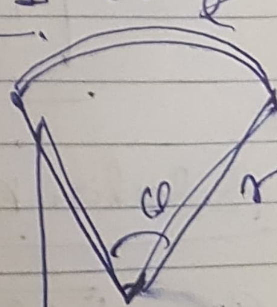
$$\boxed{1AU = 1.5 \times 10^{11} m \simeq 1.496 \times 10^{11} m}$$

(2) Light year (ly): is distance travelled by light in vacuum in 1 yr

$$V = \frac{d}{t} \rightarrow d = v \times t = [3 \times 10^{8} \times 365 \times 24 \times 60 \times 60]$$

$$\boxed{1 ly = 9.46 \times 10^{15} m}$$

(3) Parsec: is radius of a circle, at centre of which an arc of circle, 1AU long subtended an angle 1'' (second)



$$\rightarrow r = \frac{l}{\theta} = \frac{1 AU}{1 sec.}$$

$$r = \left[\frac{1.496 \times 10^{11}}{1 sec}\right] \leftarrow$$

$$\theta = \frac{l}{r}$$

$$\boxed{r = \frac{l}{\theta}}$$

$$\theta \Rightarrow 1 sec = \frac{1}{60} min = \left(\frac{1}{60} \times \frac{1}{60}\right)^{\circ} = \left(\frac{1}{60} \times \frac{1}{60} \times \frac{\pi}{180}\right) radian$$

$$\Rightarrow 1'' = \left(\frac{1}{60}\right)'$$

sec → Min → degree → Radian

$$1\,\text{Parsec} = \frac{1\,AU}{1\,sec} = \frac{1.496\times10^{11}}{\left[\frac{1}{60}\times\frac{1}{60}\times\frac{\pi}{180}\right]}$$

$$\boxed{1\,\text{Par} = 3.084\times10^{16}\,m}$$

$$\boxed{1\,\text{Par} = 3.1\times10^{16}\,m}$$

Q: Change $5''$ into Radian?

A $5'' \to \left(\frac{5}{60}\right)' = \left(\frac{5}{60}\times\frac{1}{60}\right)^{\circ} = \left(\frac{5}{60}\times\frac{1}{60}\times\frac{\pi}{180}\right)$ Radian

Note:

$$\boxed{\begin{aligned} 1\,AU &= 1.5\times10^{11}\,m \\ 1\,ly &= 9.46\times10^{15}\,m \\ 1\,\text{Parsec} &= 3.1\times10^{16}\,m \end{aligned}}$$

Relation B/w AU, ly, Parsec

(1) $$\boxed{\frac{1\,ly}{1\,AU} = \frac{9.46\times10^{15}}{1.5\times10^{11}} = 6.3\times10^{4}\,AU}$$

* $\boxed{1\,ly = 6.3 \times 10^4\,AU}$

* $\boxed{1\,Parsec = 3.26 \times ly}$

Parallax Method

↓