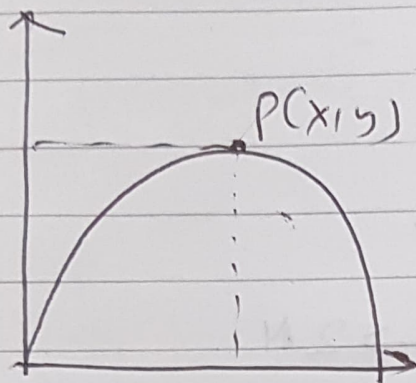


* L-10 $\rightarrow$

1

Chapter No $\rightarrow$ 4

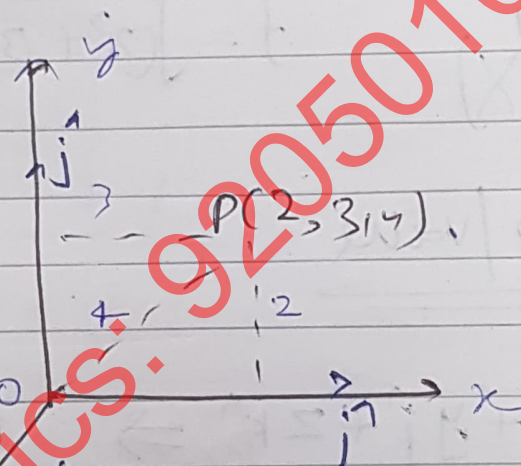
[Motion in two dimensions]



Vector

①

Interpretation



$$\vec{P} = 2\vec{i} + 3\vec{j} + 4\vec{k}$$

$$\begin{aligned} x &\rightarrow \vec{i} \\ y &\rightarrow \vec{j} \\ z &\rightarrow \vec{k} \end{aligned}$$

The vector which shows position of point P.

$$\Rightarrow \vec{P} = x\vec{i} + y\vec{j} + z\vec{k}$$

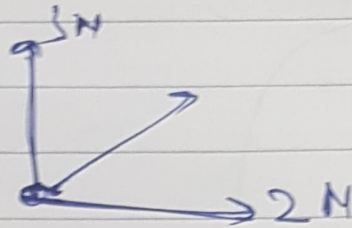
Net

$$\rightarrow f = 2N$$

$$\rightarrow \vec{f} = 2N(\hat{x})$$

Vector

$$\vec{f} = 2\hat{i} + 3\hat{j}$$



$$\vec{f} = 50N(\hat{x})$$

$$f = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

② Magnitude / value

$$\rightarrow \vec{P} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow P = |\vec{P}| = \sqrt{x^2 + y^2 + z^2}$$

$$\therefore \vec{P} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$|\vec{P}| = P$$

$$P = |\vec{P}| = \sqrt{2^2 + 3^2 + 4^2}$$

$$P = \sqrt{29}N$$

③ Direction ($\hat{p}$)

$$\rightarrow \hat{p} = \frac{\text{value p.ing}}{\text{Magnitude}}$$

$$\rightarrow \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\rightarrow \hat{p} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

Q. Find the direction of vector $\vec{A}$
which is $[\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}]$

$$\rightarrow \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j} + 2\hat{k}}{\sqrt{3^2 + 4^2 + 2^2}} = \frac{3\hat{i} + 4\hat{j} + 2\hat{k}}{\sqrt{9 + 16 + 4}}$$

$$\rightarrow \hat{A} = \frac{3\hat{i} + 4\hat{j} + 2\hat{k}}{\sqrt{29}}$$

Note.

① $f = 5N$ ✓

② $\vec{f} = 5N$ ✗

$\vec{f} = 5\hat{i}$ ✓

$\vec{f} = 5\hat{k}$ ✓

③ $\vec{f} = 5\hat{i}^T$ (✓)

⑤

$\vec{f} = 6\hat{i}$ (X)

~~$\vec{f} = 6\hat{i}$~~ vector

$\vec{f} = f = 6N$ standard

④

displacement:

$$\vec{y} = x\hat{i} + y\hat{j} + z\hat{k}$$

→ velocity:

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

→ acceleration:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

→ force:

$$\vec{f} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$$

Types of vectors

3

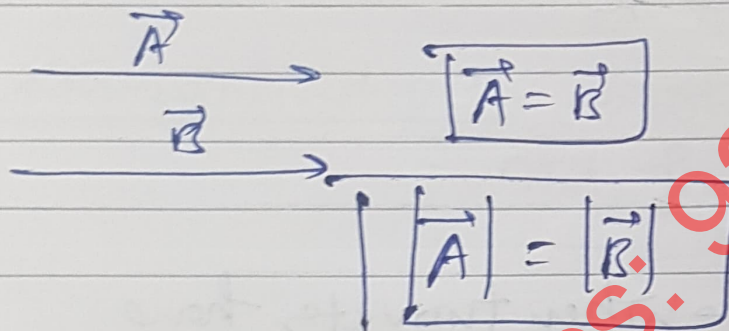
The vector

① Position vector → Which shows position of a point.

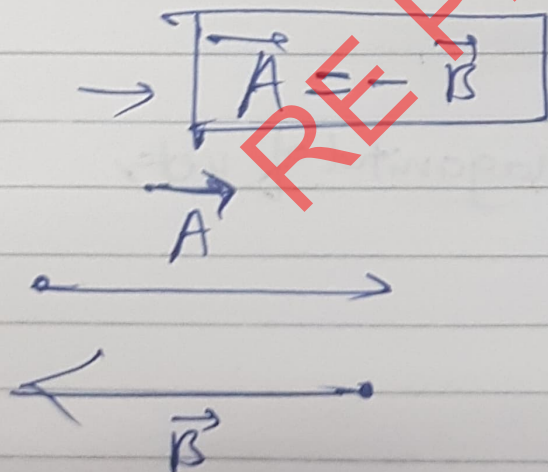
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

② Equal vector → When two vectors have same direction & magnitude they called as

"Equal vectors"

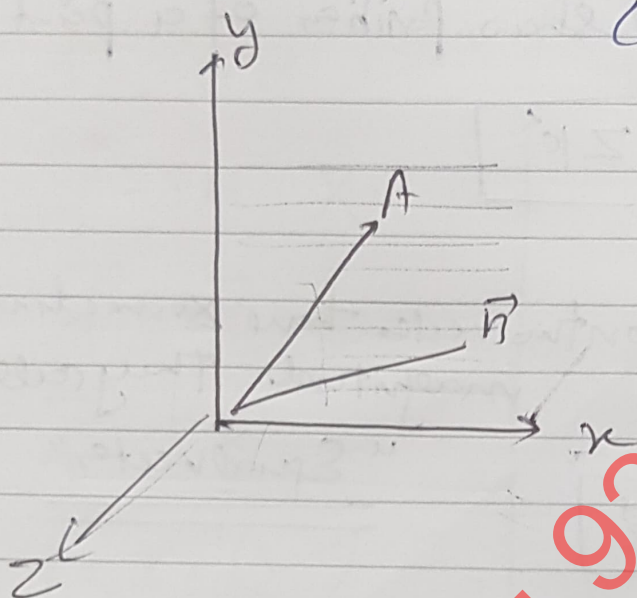

$$\vec{A} = \vec{B}$$
$$|\vec{A}| = |\vec{B}|$$

③ Negative vector → When vectors have same magnitude but opposite direction

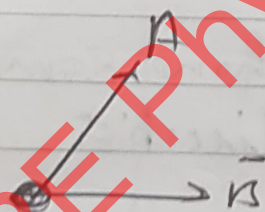

$$\vec{A} = -\vec{B}$$

Same
↓

- ④ Co-planar Vector : → When two vectors lie in same plane
called "Co-planar vector"



- ⑤ Co-initial Vector : → When two vectors have same initial point
called "Co-initial vector"



- ⑥ Zero/Null vector : → When magnitude of vector is zero

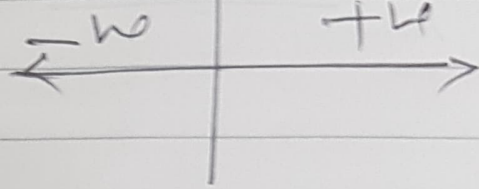
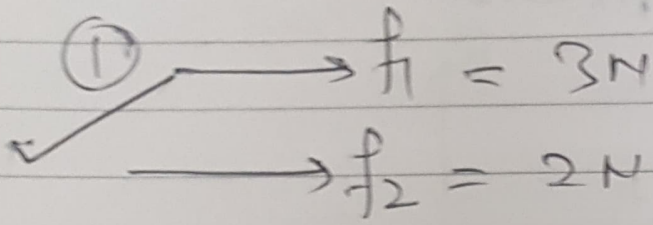
$$|\vec{A}| = A = 0$$

⑦ Localized vector →

When initial point of vector is fixed it
called "Localized Vector"

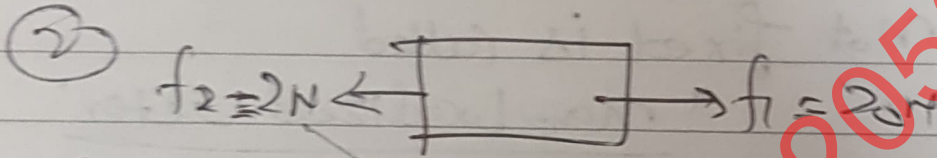
⑧ Non-localized vector → When initial point of vector is
not fixed is called
"non-localized vector"

Addition of vector

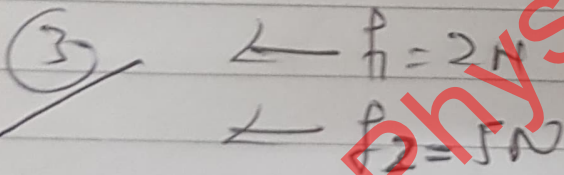


$$\rightarrow f = f_1 + f_2$$

$$\rightarrow \boxed{f = 3 + 2 = 5\text{ N}}$$

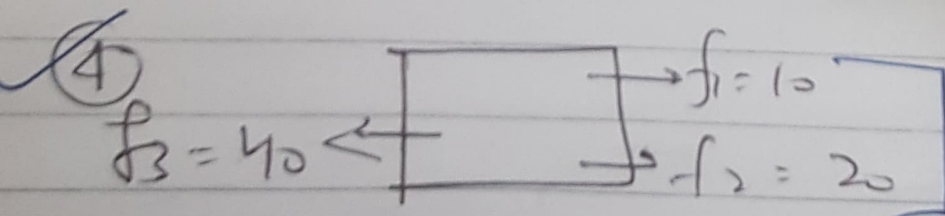


$$\rightarrow \boxed{f = 20 - 2 = 18\text{ N}}$$



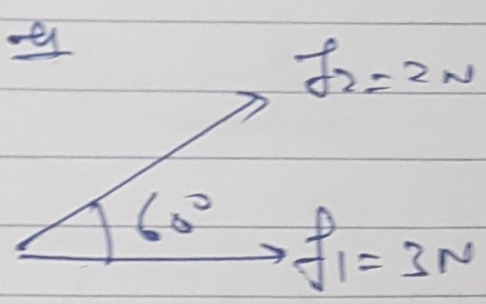
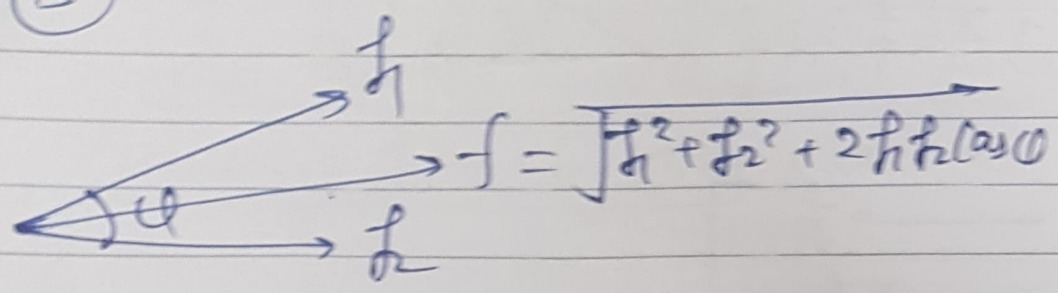
$$\rightarrow f = -f_1 - f_2$$

$$\rightarrow \boxed{f = -2 - 5 = -7\text{ N}}$$



$$\rightarrow \boxed{f_{\text{net}} = 10 + 20 - 40 = 30 - 40 = -10\text{ N}}$$

(5)



$$\rightarrow f = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \times \cos 60^\circ}$$

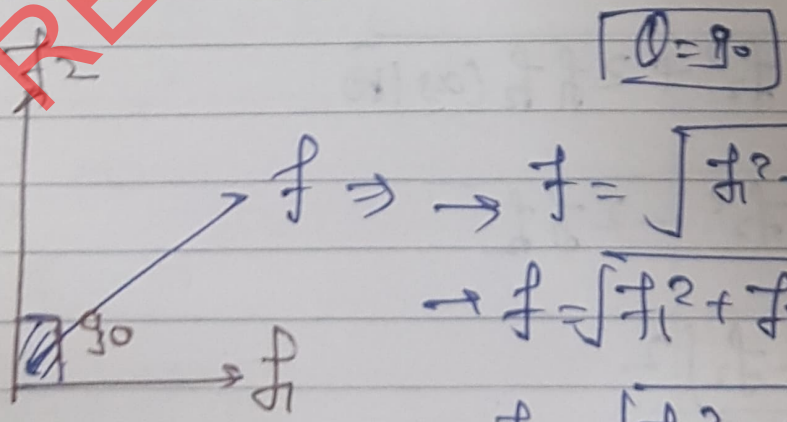
$$\rightarrow f = \sqrt{4 + 9 + 12 \times \frac{1}{2}}$$

$$\rightarrow f = \sqrt{4 + 9 + 6}$$

$$\rightarrow f = \sqrt{19} \text{ N}$$

REF Physics: 9205010851

(6)



$$\theta = 90^\circ$$

$$f \Rightarrow f = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos\theta}$$

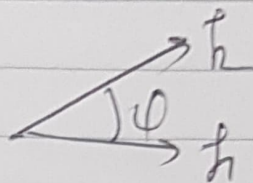
$$\rightarrow f = \sqrt{f_1^2 + f_2^2 + 2f_1f_2 \times \cos 90^\circ}$$

$$\rightarrow f = \sqrt{f_1^2 + f_2^2 + 2f_1f_2 \times 0}$$

$$\rightarrow f = \sqrt{f_1^2 + f_2^2}$$

Not

$$\rightarrow f = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos\theta}$$



(-1) $\theta = 0$ $\rightarrow f_1$
 $\rightarrow f_2$

$$\rightarrow f_M = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos 0}$$

$$\rightarrow f_M = \sqrt{f_1^2 + f_2^2 + 2f_1f_2}$$

$$\rightarrow f_M = \sqrt{(f_1 + f_2)^2}$$

$$\rightarrow \boxed{f_M = (f_1 + f_2)}$$

Maximum force

(-2) $\theta = 180^\circ$

$$[\because \cos 180^\circ = -1]$$

$$\rightarrow f_M = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos 180^\circ}$$

$$\rightarrow f_M = \sqrt{f_1^2 + f_2^2 - 2f_1f_2}$$

$$\rightarrow f_M = \sqrt{(f_1 - f_2)^2}$$

$$\rightarrow \boxed{f_M = (f_1 - f_2)}$$

Minimum force

The ~~greater~~ greatest force on A body is 50 N
 & least force on Body is 20 N. Find the
 resultant force if θ is 60° ?

A $f_1 = A$, $f_2 = B$

greater force:

$$f_1 + f_2 = 50 \text{ N} \quad (1)$$

Least force: $f_1 - f_2 = 20 \text{ N} \quad (2)$

Solve eq (1) & (2)

$$f_1 + f_2 = 50$$

$$f_1 - f_2 = 20$$

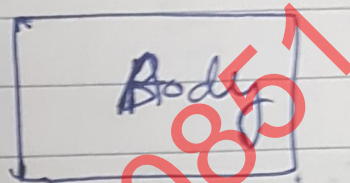
$$2f_1 = 50 + 20$$

$$f_1 = \frac{70}{2} = 35 \text{ N}$$

$$\rightarrow f_1 + f_2 = 50$$

$$\rightarrow 35 + f_2 = 50$$

$$f_2 = 15$$



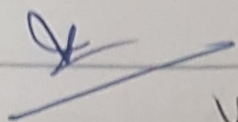
$$F_N = \sqrt{f_1^2 + f_2^2 + 2f_1f_2 \cos \theta}$$

$$= \sqrt{(35)^2 + (15)^2 + 2(35)(15)\left(\frac{1}{2}\right)}$$

$$= \sqrt{1225 + 225 + 525}$$

$$F_N = \sqrt{1975}$$

$$F_N = \sqrt{1975} \text{ N}$$

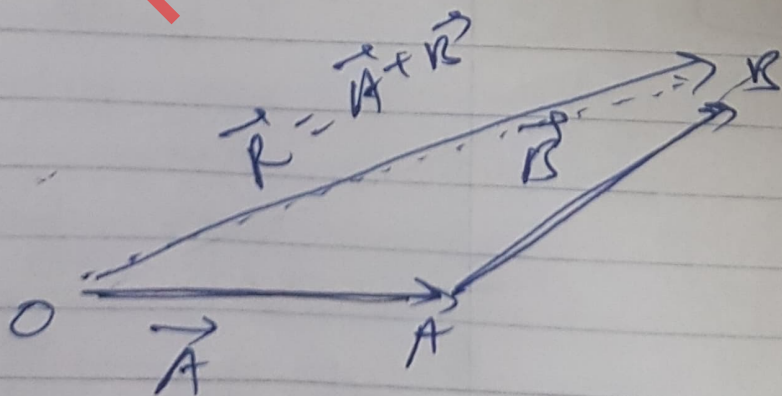
Vector Addition

* ① Triangle Law of Addition

② Parallelogram Law of Addition

* Triangle Law of Vector Addition :

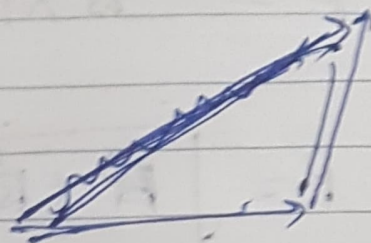
It states that If two vectors acting on a particle at the same time are represented in magnitude & direction by the two sides of Triangle taken in one order, their resultant vector is represented in magnitude & direction by third side of Triangle taken in opposite order.



Let $\vec{OA}$ is one vector $\rightarrow \vec{A}$
 Let $\vec{AB}$ is other vector $\rightarrow \vec{B}$

$$\vec{R} = \vec{A} + \vec{B}$$

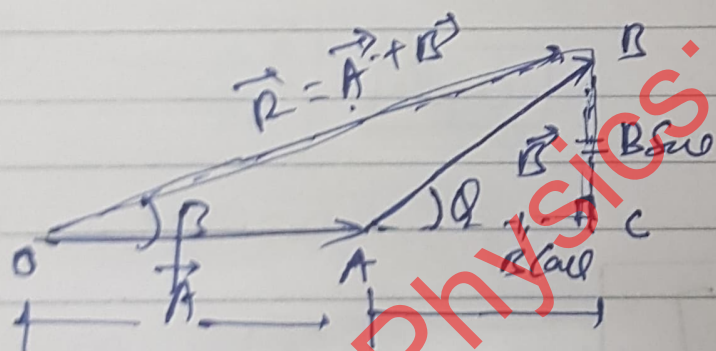
Let $\vec{R}$ is Resultant vector



$$\vec{r} = (5\hat{i}) + (6\hat{j})$$

$$|\vec{r}| = r = \sqrt{5^2 + 6^2}$$

$$= \sqrt{25 + 36}$$



Let BC is perpendicular to OC
 In $\triangle OBC$

$$OB^2 = BC^2 + OC^2$$

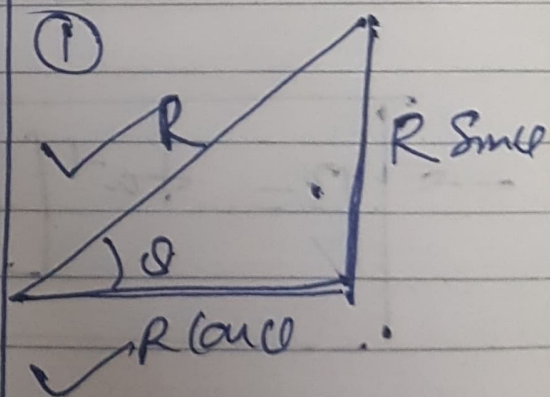
$$R^2 = (B \sin \theta)^2 + (A + B \cos \theta)^2$$

$$R^2 = B^2 \sin^2 \theta + A^2 + B^2 \cos^2 \theta + 2AB \cos \theta$$

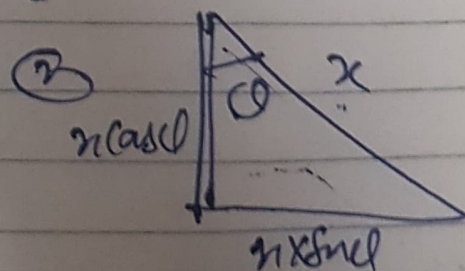
$$R^2 = B^2 [\sin^2 \theta + \cos^2 \theta] + A^2 + 2AB \cos \theta$$

Proof

①



②



$$R^2 = B^2 \times 1 + A^2 + 2AB \cos \theta$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

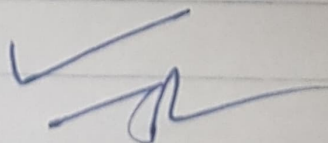
$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

direction

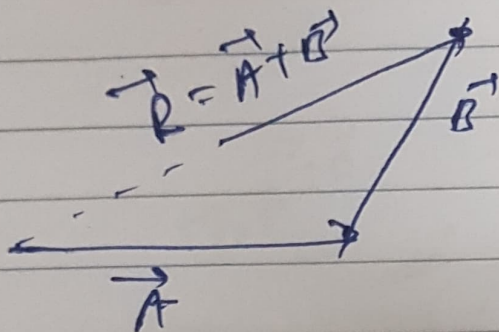
$$\rightarrow \tan \beta = \frac{BC}{OC} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\rightarrow \tan \beta = \left[\frac{B \sin \theta}{A + B \cos \theta} \right]$$

$$\beta = \tan^{-1} \left[\frac{B \sin \theta}{A + B \cos \theta} \right]$$



Note



① $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

② $\theta = \tan^{-1} \left[\frac{B \sin \theta}{A + B \cos \theta} \right]$

③ $\begin{matrix} \longrightarrow \vec{A} \\ \longrightarrow \vec{B} \end{matrix} \quad \boxed{\theta = 0}$

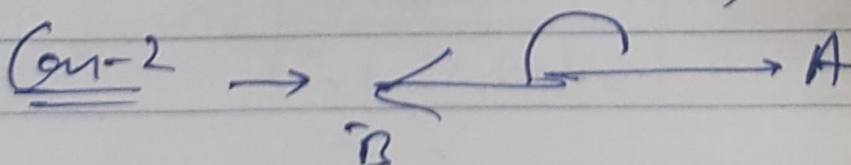
Case 1: $\cos 0 = 1$

$$R = \sqrt{A^2 + B^2 + 2AB \times 1}$$

$$R = \sqrt{A^2 + B^2 + 2AB} = \sqrt{(A+B)^2}$$

$$\boxed{R_{\text{max}} = A + B}$$

$180^\circ = \theta$



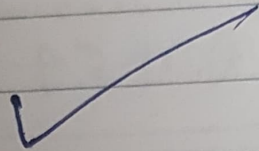
Case-2 $\cos 180 = -1$

$$R = \sqrt{A^2 + B^2 + 2AB \cos(-1)}$$

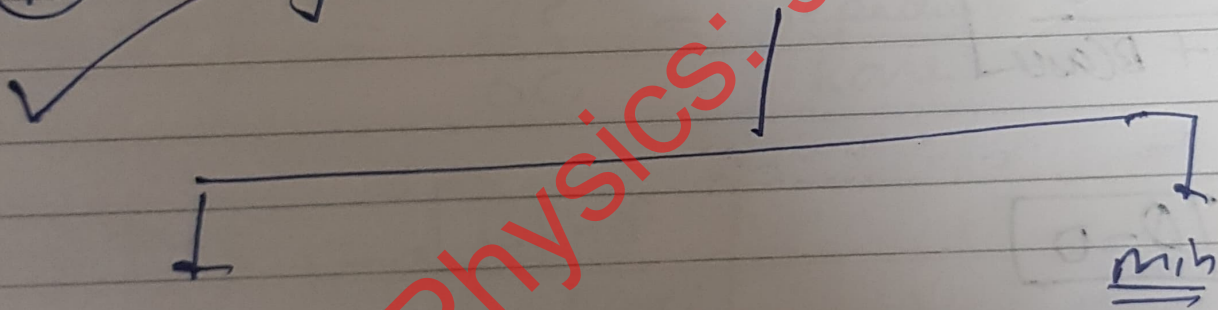
$$R = \sqrt{A^2 + B^2 - 2AB}$$

$$R = \sqrt{(A-B)^2}$$

$$\boxed{R_{\min} = (A-B)}$$



④ $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$



Max
 $\boxed{\theta = 0}$

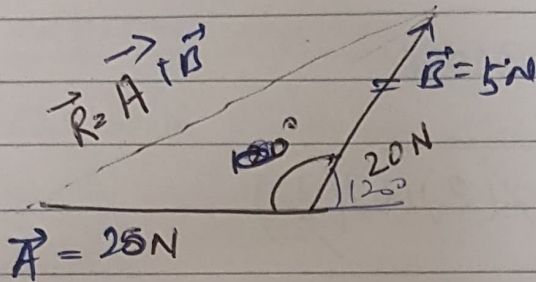
min
 $\boxed{\theta = 180}$

$$\boxed{R_{\max} = A + B}$$

$$\boxed{R_{\min} = (A-B)}$$

Q Two force 20N & 5N are acting at angle 120° between them. find the Resultant force in Magnitude & direction

A



(4) $\rightarrow R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

$\rightarrow R = \sqrt{(20)^2 + (5)^2 + 2(20)(5)\left(\frac{-1}{2}\right)}$

$\rightarrow R = \sqrt{400 + 25 - 100}$

$\rightarrow R = \sqrt{325} \text{ N}$

(5) direction

$\tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$

$\tan \beta = \frac{5 \sin 120}{20 + 5 \cos 120} = \frac{5 \times \frac{\sqrt{3}}{2}}{20 + 5 \times \left(-\frac{1}{2}\right)} = \frac{5 \times \frac{\sqrt{3}}{2}}{20 - \frac{5}{2}} = \frac{5 \times \frac{\sqrt{3}}{2}}{\frac{15}{2}}$

$\tan \beta = \frac{\sqrt{3}}{3}$

Q: Let two forces 8N & 6N are acting at $\theta = 60^\circ$

Find the (a) Magnitude
(b) direction.

A

(a) $\vec{R} = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

$$= \sqrt{(8)^2 + (6)^2 + 2(8)(6)\left(\frac{1}{2}\right)}$$

$$= \sqrt{64 + 36 + 48}$$

$$\boxed{\vec{R} = \sqrt{148} \text{ N}}$$

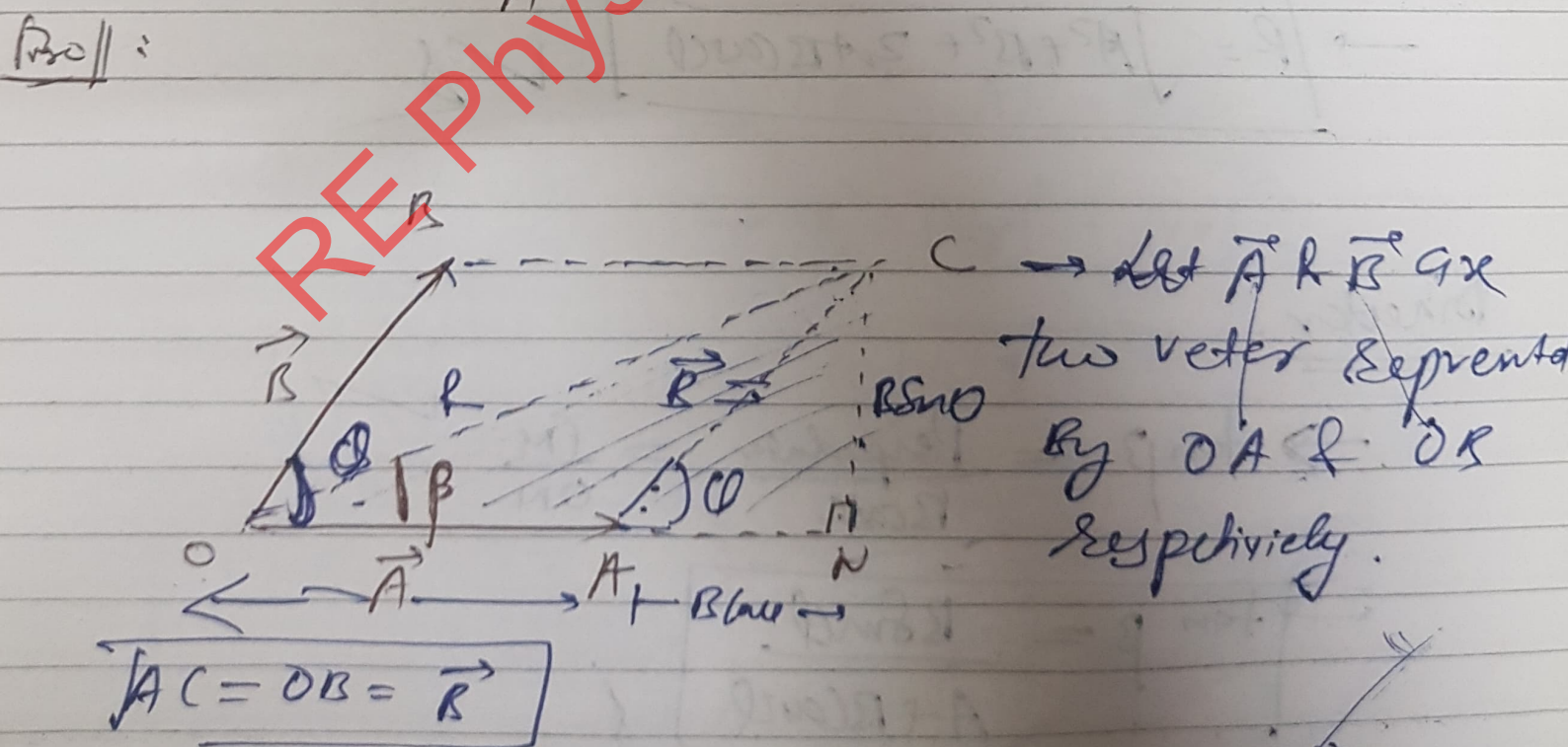
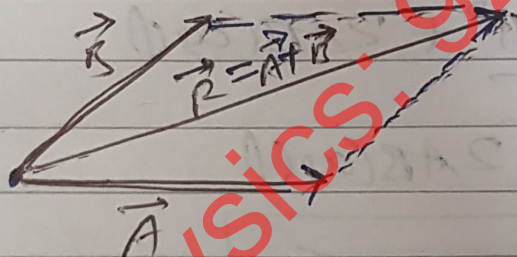
(b) $\tan \beta = \frac{B \sin \theta}{A + B \cos \theta} = \frac{6 \times \frac{\sqrt{3}}{2}}{8 + 6 \times \frac{1}{2}} = \frac{3\sqrt{3}}{11}$

$$= \frac{3\sqrt{3}}{11}$$

③

* Parallelogram Law of vector Addition

It states that If two vectors acting on a particle at the same time by represented in magnitude & direction by two adjacent sides of a parallelogram drawn from a point their resultant vector is represented in magnitude & direction by diagonal of parallelogram drawn from the same point.



Qm ON

$$\rightarrow OC^2 = CH^2 + (ON)^2$$

$$\rightarrow R^2 = (R \sin \theta)^2 + (A + R \cos \theta)^2$$

$$\rightarrow R^2 = \underline{R^2 \sin^2 \theta} + A^2 + \underline{R^2 \cos^2 \theta} + 2AB \cos \theta$$

$$\rightarrow R^2 = R^2 [\underline{\sin^2 \theta + \cos^2 \theta}] + A^2 + 2AB \cos \theta$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\rightarrow R^2 = R^2 \times 1 + A^2 + 2AB \cos \theta$$

$$\rightarrow R^2 = A^2 + R^2 + 2AB \cos \theta$$

$$\rightarrow \boxed{R = \sqrt{A^2 + R^2 + 2AB \cos \theta}} \quad \checkmark$$

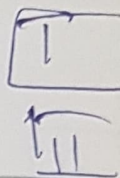
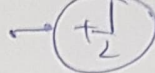
Director

$$\rightarrow \tan \beta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CM}{ON} \quad \checkmark$$

$$\rightarrow \boxed{\tan \beta = \frac{R \sin \theta}{A + R \cos \theta}}$$

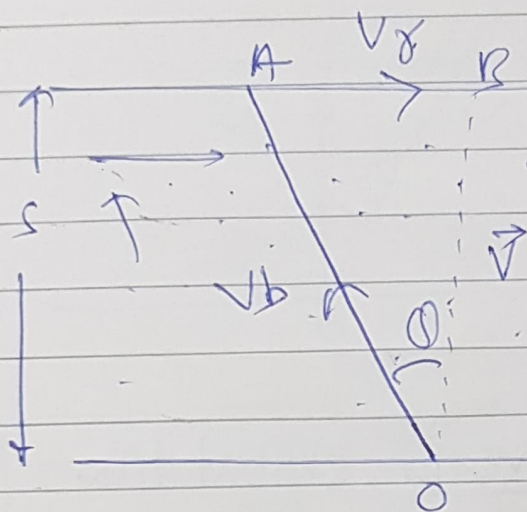
Ind:

L-13



River & Boats
↓

C-1: When a boat tends to cross a river in shortest ~~the~~ path
for covered in shortest time
It will move in opposite



direction of
river

Let 's' is width of river

Let V_b is velocity of Boat

Let V_r is velocity of river

$$\rightarrow OA^2 = AB^2 + BO^2$$

$$\rightarrow V_b^2 = (V_r)^2 + (V_o)^2$$

$$\rightarrow V = \sqrt{V_b^2 + V_r^2}$$

$$\rightarrow \tan \theta = \frac{AB}{BO}$$

$$\rightarrow \tan \theta = \frac{V_r}{V_b}$$

$$\tan \theta = \frac{V_r}{\sqrt{V_b^2 + V_r^2}}$$

time taken to covered

$$V = \frac{d}{t}$$

$$t = \frac{d}{V} = \frac{S}{V_b}$$

$\rightarrow V_r = 5 \text{ m/s}$

$\rightarrow V_b = 10 \text{ m/s}$

Q. Boat want to cross river in shortest time
By which it have ~~some~~ velocity S_0 , it cross in
shortest ~~time~~ path?

A. $V = \sqrt{V_b^2 - V_r^2}$

$\rightarrow V = \sqrt{(10)^2 - (5)^2}$

$V = \sqrt{100 - 25}$

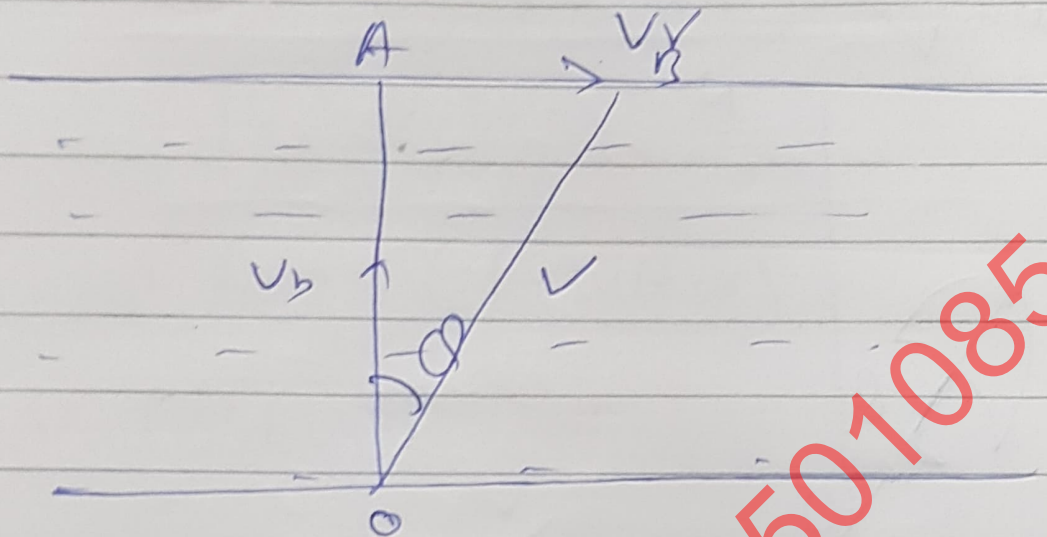
$\rightarrow V = \sqrt{75} = \sqrt{25 \times 3}$

$V = 5\sqrt{3}$

$V = 5 \times 1.73 = 8.85$

in opposite dir.

(-2) : When a boat tends to cross a river in shortest time :



$$\tan \theta = \frac{AB}{OA} = \frac{v_r}{v_b}$$

Resultant Velocity

$$\rightarrow OA^2 + AB^2 = OB^2$$

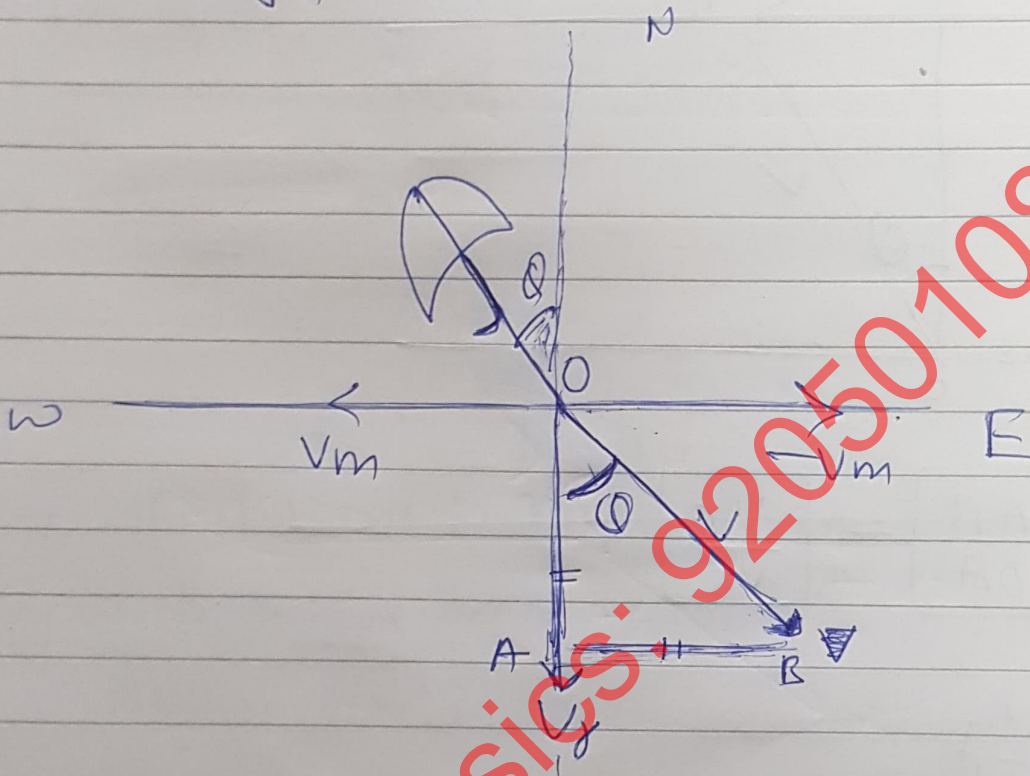
$$\rightarrow v_b^2 + v_r^2 = v^2$$

$$v = \sqrt{v_b^2 + v_r^2}$$

Relative velocity of rain w.r.t to moving Man

Let $V_m \rightarrow$ Velocity of man ✓

Let $V_r \rightarrow$ velocity of rain. ✓



In ΔOAB

$$\rightarrow \tan \theta = \frac{AB}{OA}$$

$$\rightarrow \tan \theta = -\frac{V_m}{V_r}$$

$$\rightarrow \boxed{\tan \theta = -\frac{V_m}{V_r}}$$

$$\rightarrow V_{rm} = \sqrt{(V_r)^2 + (-V_m)^2 + 2V_r(-V_m)\cos \theta}$$

$$\boxed{V_{rm} = \sqrt{V_r^2 + V_m^2 - 2V_r V_m \cos \theta}}$$

Note

① shortest path

(Resultat)

$$V = \sqrt{V_b^2 - V_r^2}$$

② shortest time

(Resultat)

$$V = \sqrt{V_b^2 + V_r^2}$$

③ Rain & Man

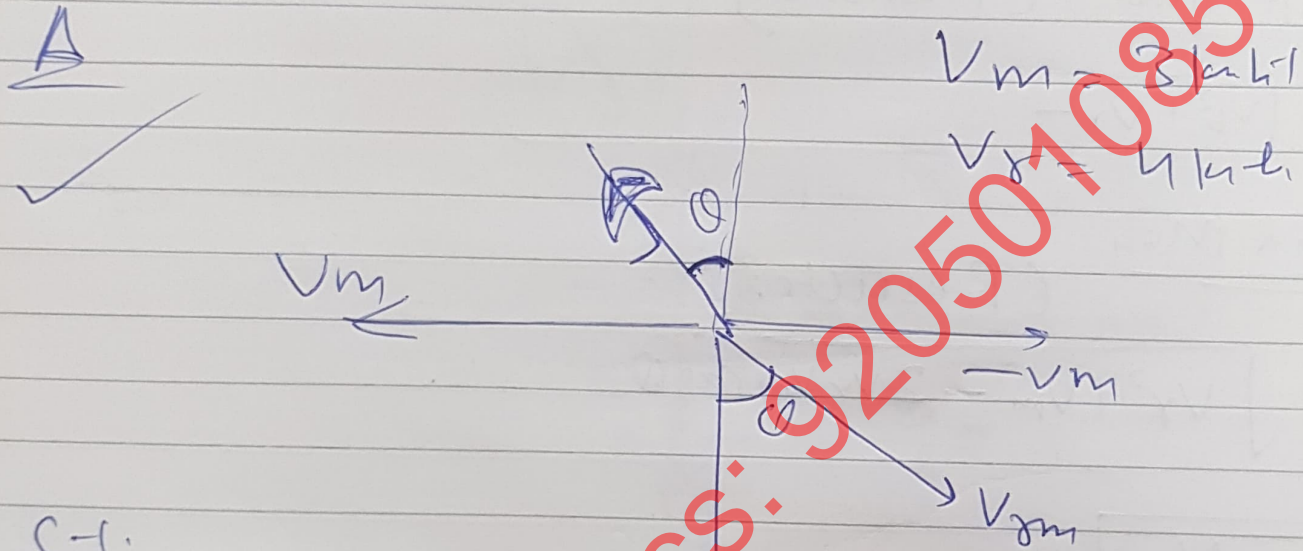
(Resultat)

$$V_{\text{rms}} = \sqrt{V_r^2 + V_m^2 - 2V_r V_m \cos \theta}$$

$$\tan \theta = \frac{V_r}{V_m}$$

Q: A man is walking on level road at speed of 8 km/h. Rain drops fall vertically with velocity 4 km/h.

Find the velocity of rain drop with respect to man, in which direction man should hold his umbrella to protect himself?



C-1:

$$\rightarrow \tan \theta = \frac{V_m}{V_r} = \frac{8}{4} \rightarrow \tan \theta = 2$$

$\theta = \tan^{-1} 2$ $\rightarrow$ $\theta = 63.4^\circ$ $\theta = 45^\circ$ ✓
 $\theta \neq 2$

C-2 $\rightarrow V_{rm} = \sqrt{V_r^2 + V_m^2 - 2V_r V_m \cos \theta}$

$$V_{rm} = \sqrt{(4)^2 + (8)^2 - 2 \times 4 \times 8 \times \cos 45^\circ}$$

$$V_m = \sqrt{16 + 16 - 32 \times 1} \sqrt{2}$$

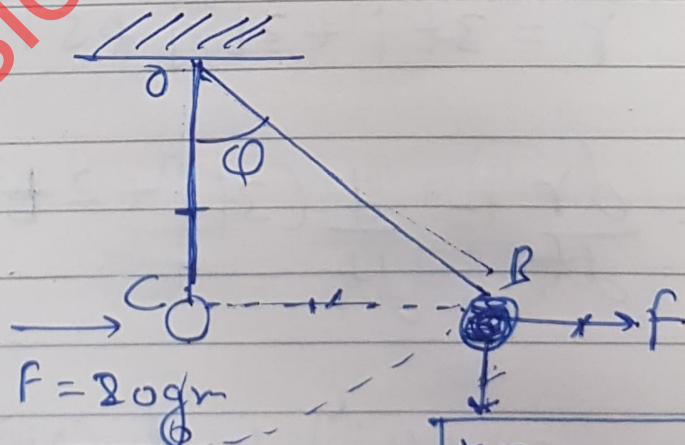
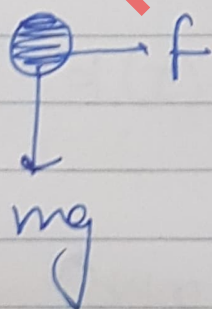
$$V_m = \sqrt{(32 - 32 \times 1) \sqrt{2}}$$

✓

Q: A bob weighing 50 gsm hangs vertically at the end of string 50 cm long. If 20 gsm force is applied horizontally by how much distance the bob is pulled aside from its initial position when it reached in equilibrium position?

✓ $m = 50 \text{ gm}$
 $l = 50 \text{ cm} = 0.50 \text{ m}$

* $g = 9.8 \approx 10$



$$F = 20 \text{ gm}$$

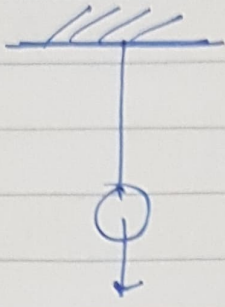
$$F = (20 \times 980) \text{ N}$$

$$mg = \text{weight}$$

$$\frac{F}{BC} = \frac{mg}{OC}$$

$$\rightarrow \frac{20 \times 980}{50 BC} = \frac{50 \times 10^{-3} \times 10}{0.50} \rightarrow BC = ?$$

Note:



$$\left[\frac{d(x^y)}{dx} = 5x^7 \right]$$

$$\rightarrow W = m \times g = \text{weight}$$

$$\rightarrow [g = 9.8 \approx 10]$$

Q: The position of particle is $\vec{r}$

$$\vec{r} = 3t \hat{i} + 2t^2 \hat{j} + 5\hat{k}$$

Find the velocity at $t = 2 \text{ sec}$?

A $\vec{r} = 3t \hat{i} + 2t^2 \hat{j} + 5\hat{k}$

$$\rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} (3t \hat{i} + 2t^2 \hat{j} + 5\hat{k})$$

$$\rightarrow \vec{v} = 3 \frac{d(t \hat{i})}{dt} + \frac{d(2t^2)}{dt} \hat{j} + \frac{d(5)}{dt} \hat{k}$$

$$\rightarrow \vec{v} = (3 \times 1 \times) \hat{i} + (2 \times 2 \times t) \hat{j} + 5 \times 0 \times \hat{k}$$

$$\rightarrow \boxed{\vec{v} = 3\hat{i} + 4t\hat{j}}$$

at $t = 2 \text{ s}$

$$\vec{V} = 3\hat{i} + 4 \times 2 \hat{j}$$

$$\Rightarrow \boxed{\vec{V} = (3)\hat{i} + (8)\hat{j}}$$

Magnitude:

$$\left. \begin{array}{l} v_x = 3 \\ v_y = 8 \end{array} \right\}$$

$$V = \sqrt{3^2 + 8^2}$$

$$V = \sqrt{9 + 64}$$

$$V = \sqrt{73}$$

$$\frac{d(4t)}{dt} = 5 + 4$$

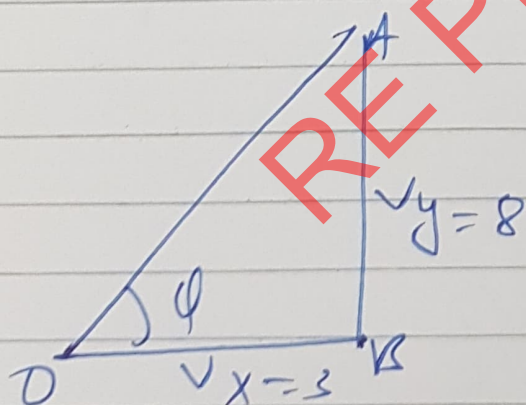
$$\frac{d(t^2)}{dt} = 2t$$

$$\frac{d(5)}{dt} = 0$$

$$\frac{d(t)}{dt} = 1$$

$$\frac{d(6 + 8)}{dt} = 6 \times 8 \times 1$$

©: If θ is angle which the direction of $\vec{v}$ makes the x-axis



$$\frac{PR}{HR} = \frac{P}{H}$$

$$\rightarrow \tan \theta = \frac{AB}{OB} = \frac{8}{3} \rightarrow$$

$$\boxed{\theta = \tan^{-1} \left[\frac{8}{3} \right]}$$

[Signature]

Q $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

(c) A

$$A = |\vec{A}| = \sqrt{(2)^2 + (4)^2 + (-5)^2}$$

$$\rightarrow A = \sqrt{4 + 16 + 25}$$

$$\rightarrow A = \sqrt{20 + 25}$$

$$\rightarrow A = \sqrt{45}$$

(b) direction cosine of vector $\vec{A}$

$$\rightarrow \cos \alpha = \frac{A_x}{A} = \frac{4}{\sqrt{45}}$$

$$\rightarrow \cos \beta = \frac{A_y}{A} = \frac{4}{\sqrt{45}}$$

$$\rightarrow \cos \gamma = \frac{A_z}{A} = \frac{-5}{\sqrt{45}}$$

S

Note:

→ $\vec{V} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ & direction cosine of $\vec{V}$. 16

(a) $V_x = 4$
 $V_y = 5$
 $V_z = 6$

$$\cos \alpha = \frac{Ax}{A} = \frac{4}{\sqrt{77}}$$
$$\cos \beta = \frac{Ay}{A} = \frac{5}{\sqrt{77}}$$
$$\cos \gamma = \frac{Az}{A} = \frac{6}{\sqrt{77}}$$

Magnitude

(b) $|\vec{V}| = |\vec{V}| = V = \sqrt{4^2 + 5^2 + 6^2} = \sqrt{16 + 25 + 36} = \sqrt{77}$

(c) direction

$$\hat{V} = \frac{4\hat{i} + 5\hat{j} + 6\hat{k}}{\sqrt{4^2 + 5^2 + 6^2}} = \frac{4\hat{i} + 5\hat{j} + 6\hat{k}}{\sqrt{77}}$$

Product of vectors

✓ Scalar Product
[dot Product]

* Cross Product
[x]

Let A & B are two vectors

Let

$$\begin{aligned}\vec{A} &= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\ \vec{B} &= B_x \hat{i} + B_y \hat{j} + B_z \hat{k}\end{aligned}$$

* Dot Product / Scalar Product:

$$\Rightarrow \vec{A} \cdot \vec{B} = AB \cos \theta$$

Let: $\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (A_x B_x) \hat{i} \cdot \hat{i} + (A_x B_y) \hat{i} \cdot \hat{j} + (A_x B_z) \hat{i} \cdot \hat{k} + \\ &+ (A_y B_x) \hat{j} \cdot \hat{i} + (A_y B_y) \hat{j} \cdot \hat{j} + (A_y B_z) \hat{j} \cdot \hat{k} + \\ &+ (A_z B_x) \hat{k} \cdot \hat{i} + (A_z B_y) \hat{k} \cdot \hat{j} + \underline{(A_z B_z) \hat{k} \cdot \hat{k}}\end{aligned}$$

$$\rightarrow \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\rightarrow [\hat{j} \cdot \hat{j} = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0]$$

Ans

$$\vec{A} \cdot \vec{B} = (\underline{A_x B_x})1 + (A_x B_y)0 + (A_x B_z)0 + (A_y B_x)0 + (\underline{A_y B_y})1 + (A_y B_z)0 + (A_z B_x)0 + (A_z B_y)0 + (\underline{A_z B_z})1$$

$$= (A_x B_x) + 0 + 0 + 0 + (A_y B_y) + 0 + 0 + 0 + (A_z B_z)$$

$$\boxed{\vec{A} \cdot \vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)} \quad \underline{A}$$

Q: If $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ &

$$\vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}$$

find $\vec{A} \cdot \vec{B}$ or find the dot product or find the scalar product

Ans $\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})(B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j} + 4\hat{k})(3\hat{i} + 5\hat{j} + 6\hat{k})$$

$$\vec{A} \cdot \vec{B} = (2 \times 3) + (3 \times 5) + (4 \times 6)$$

$$\vec{A} \cdot \vec{B} = 6 + 15 + 24 = \underline{45}$$

Q:

$$\vec{F} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\vec{S} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

Find the work done?

Find Power for 2 sec

Ans

① $\rightarrow \boxed{W = \vec{F} \cdot \vec{S}}$

$$\rightarrow W = (2\hat{i} + 4\hat{j} + 6\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 5\hat{k})$$

$$\rightarrow W = 2 \times 3 + 4 \times 2 + 6 \times 5$$

$$\rightarrow W = 6 + 8 + 30$$

$$\rightarrow W = 44$$

$$\boxed{W = 44 \text{ J}}$$

② $P = \frac{W}{t}$

$$P = \frac{44}{2}$$

$$\boxed{P = 22 \text{ watt}}$$

Q: If $\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ ✓
 & $\vec{v} = 2\hat{i} + 5\hat{j} + 6\hat{k}$ ✓

Find the Power?

△ $\boxed{P = \vec{F} \cdot \vec{v}}$

$$\rightarrow P = (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (2\hat{i} + 5\hat{j} + 6\hat{k})$$

$$P = 2 \times 2 + 3 \times 5 + 4 \times 6$$

$$P = 4 + 15 + 24$$

$$P = 43 \text{ watt}$$

$$\boxed{P = \frac{W}{t} = \vec{f} \cdot \frac{d}{dt}}$$

$$P = \vec{f} \cdot \vec{v}$$

NOTE

$$\textcircled{1} W = \vec{f} \cdot \vec{v}$$

$$\textcircled{2} P = \vec{f} \cdot \vec{v}$$

Q. Are the Magnitude and direction of $(\vec{A} - \vec{B})$ same as that of $(\vec{B} - \vec{A})$?

$$A. \quad \vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

They have same Magnitude But opposite direction

Q. What is the property of two vectors if

$$\vec{A} + \vec{B} = \vec{A} - \vec{B}$$

$$A. \quad \vec{A} + \vec{B} = \vec{A} - \vec{B} \rightarrow \vec{B} + \vec{B} = 0 \rightarrow 2\vec{B} = 0$$

$$\boxed{2\vec{B} = 0}$$

Null vector

N.H.

$$\boxed{\vec{A} \cdot \vec{B} = AB \cos \theta}$$

Q If $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
 $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

Find the angle θ between them?

A

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\boxed{\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}}$$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= 2 \times 1 + 3 \times 2 + 4 \times 3 \\ &= 2 + 6 + 12 \\ &= 20\end{aligned}$$

$$\begin{aligned}|\vec{A}| &= \sqrt{(2)^2 + (3)^2 + (4)^2} \\ &= \sqrt{4 + 9 + 16} = \sqrt{29}\end{aligned}$$

$$|\vec{B}| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

putting values

$$\cos \theta = \frac{20}{\sqrt{29} \times \sqrt{14}}$$

$$\Rightarrow \theta = \cos^{-1} \left[\frac{20}{\sqrt{29} \sqrt{14}} \right]$$

Test → 2

L-3 * L-4 L-5, (L-6)

~~Product of vector~~ (L-6)

Product of vector

→ cross product → $\vec{A} \times \vec{B} = AB \sin \theta$

$$\text{Let } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

Q. $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 6\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{A} \cdot \vec{B} = [3 \times 6 + 4 \times 6 + 5 \times 3] = [18 + 24 + 15] = \underline{57}$

$\vec{A} \times \vec{B}$

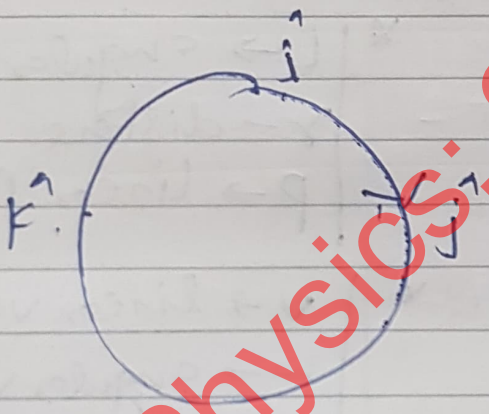
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 6 & 6 & 3 \end{vmatrix}$$

$\vec{A} \times \vec{B} = \left[\hat{i}(4 \times 3 - 6 \times 5) - \hat{j}(3 \times 3 - 6 \times 5) + \hat{k}(3 \times 6 - 6 \times 4) \right]$
 $= \hat{i}(12 - 30) - \hat{j}(9 - 30) + \hat{k}(18 - 24)$
 $= \hat{i}(-18) - \hat{j}(-21) + \hat{k}(6)$

$\boxed{\vec{A} \times \vec{B} \rightarrow -18\hat{i} + 21\hat{j} - 6\hat{k}}$

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= \underline{(A_x B_y)} \hat{i} \times \hat{j} + (A_x B_z) (\hat{i} \times \hat{k}) + (A_y B_x) \hat{j} \times \hat{i} + \underline{A_y B_y} \hat{j} \times \hat{j} + A_y B_z (\hat{j} \times \hat{k}) + (A_z B_x) \hat{k} \times \hat{i} + (A_z B_y) \hat{k} \times \hat{j} + \underline{(A_z B_z)} \hat{k} \times \hat{k}$$



①

$\hat{i} \times \hat{j} = \hat{k}$	$\hat{j} \times \hat{i} = -\hat{k}$
$\hat{j} \times \hat{k} = \hat{i}$	$\hat{k} \times \hat{j} = -\hat{i}$
$\hat{k} \times \hat{i} = \hat{j}$	$\hat{i} \times \hat{k} = -\hat{j}$

② $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$$\vec{A} \times \vec{B} = (A_x B_y) \times 0 + (A_x B_z) \hat{k} + (A_y B_z) (-\hat{j}) + (A_y B_x) (-\hat{k}) + A_y B_y (0) + A_y B_z (\hat{i}) + (A_z B_x) (\hat{j}) + A_z B_y (-\hat{i}) + A_z B_z \times (0)$$

A

$$\vec{A} \times \vec{B} = A_x B_y (\hat{k}) + A_x B_z (\hat{j}) + A_y B_x (-\hat{k}) + A_y B_z (\hat{i}) + A_z B_x (\hat{j}) + A_z B_y (-\hat{i})$$

K

Notes

① ✓

$$\vec{L} = \vec{r} \times \vec{F}$$

$$\begin{array}{|l} \vec{L} \rightarrow \text{Torque} \\ \vec{r} \rightarrow \text{distance} \\ \vec{F} \rightarrow \text{force} \end{array}$$

② ✓

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\begin{array}{|l} \vec{L} \rightarrow \text{Angular Momentum} \\ \vec{r} \rightarrow \text{distance} \\ \vec{p} \rightarrow \text{linear momentum} \end{array}$$

③ ✓

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\begin{array}{|l} \vec{v} \rightarrow \text{linear velocity} \\ \vec{\omega} \rightarrow \text{angular velocity} \\ \vec{r} \rightarrow \text{distance} \end{array}$$

④ ✓

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

$$\begin{array}{|l} \vec{a} \rightarrow \text{acceleration} \\ \vec{\alpha} \rightarrow \text{angular acceleration} \\ \vec{r} \rightarrow \text{distance} \end{array}$$

⑤ ✓

$$\vec{a}_c = \vec{\omega} \times \vec{v}$$

$$\begin{array}{|l} \vec{a}_c \rightarrow \text{centripetal acceleration} \\ \vec{\omega} \rightarrow \text{angular velocity} \\ \vec{v} \rightarrow \text{linear velocity} \end{array}$$

Q

If force on Body is $\vec{F}$

$$\vec{F} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{r} = \hat{i} - \hat{j} + 5\hat{k}$$

& $\vec{r}$ is perpendicular distance

then find Torque?

A

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{r} = \hat{i} - \hat{j} + 5\hat{k}$$

$$\vec{F} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F} =$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 5 \\ 2 & 3 & -4 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}(-1 \times -4 - 5 \times 3) - \hat{j}(1 \times -4 - 5 \times 2) + \hat{k}(1 \times 3 - 2 \times -1)$$

$$\vec{\tau} = \hat{i}(-4 - 15) - \hat{j}(-4 - 10) + \hat{k}(3 + 2)$$

$$\vec{\tau} = \hat{i}(-11) - \hat{j}(-14) + \hat{k} \times 5$$

$$\boxed{\vec{\tau} = -11\hat{i} + 14\hat{j} + 5\hat{k}}$$

K

Q: Find the singular Momentum if

$$\vec{r} = 5\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\vec{p} = 3\hat{i} + 2\hat{j} - \hat{k}$$

A

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

$$\vec{L} = r \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 5 & -6 & 3 \\ p & 3 & 2 & -1 \end{vmatrix}$$

$$\vec{L} = \hat{i} (-6 \times -1 - 3 \times 2) - \hat{j} (5 \times -1 - 3 \times 3) + \hat{k} (5 \times 2 + 3 \times 6)$$

$$= \hat{i} (6 - 6) - \hat{j} (-5 - 9) + \hat{k} (10 + 18)$$

$$\boxed{\vec{L} = \hat{i} + 14\hat{j} - 8\hat{k}}$$

A

2

$$\vec{A} \cdot \vec{B} = AB \cos \theta \rightarrow \underline{\text{Parallel}}$$

$$\vec{A} \times \vec{B} = AB \sin \theta \rightarrow \text{Perpendicular}$$

Q: A & B are parallel

(i) $\vec{A} \cdot \vec{B} \neq 0$

(ii) $\vec{A} \times \vec{B} = 0$

Q: A & B are perpendicular

(i) $\vec{A} \cdot \vec{B} = 0$

(ii) $\vec{A} \times \vec{B} \neq 0$

Q: If $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = 2\hat{i} + x\hat{j} + 5\hat{k}$

Both are perpendicular to each other, find value of x ?

$\vec{A} \cdot \vec{B} = 0$

$$2 \times 2 + 3x + 4(-5) = 0$$

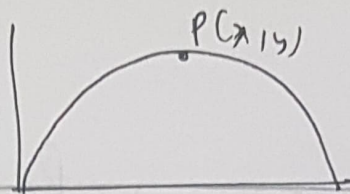
$$4 + 3x - 20 = 0$$

$$3x = 20 - 4 = 16$$

$$x = 16/3 \checkmark$$

* Projectile:

Motion of a body in 2-dimension, called "Projectile motion"



There are (2) type of Projectile

✓ (1) Horizontal Projectile

✓ (2) Vertical Projectile

Horizontal Projectile

Let at 'P'

Then one two velocity (v_x) & (v_y)

Let 'u' is initial velocity

$$u_x = u, a_x = 0$$

$$u_y = 0, a_y = a$$

Let's:

X-axis: $\rightarrow S = ut + \frac{1}{2}at^2$

$$\rightarrow X = u_x t + \frac{1}{2}a_x t^2$$

$$\rightarrow X = u_x t + \frac{1}{2} \times 0 \times t^2$$

$$\boxed{X = u_x t} \rightarrow \boxed{t = \frac{X}{u}} \quad \checkmark$$

Y-axis: $\rightarrow S = ut + \frac{1}{2}at^2$

$$\rightarrow y = u_y t + \frac{1}{2}a_y t^2$$

$$\rightarrow y = 0 \times t + \frac{1}{2} \times a \times t^2$$

$$y = \frac{1}{2} a t^2$$

$$y = \frac{1}{2} \times a \left[\frac{x}{u} \right]^2$$

$$\boxed{y = \left[\frac{a}{2u^2} \right] x^2 = kx^2}$$

$$x_{10} =$$

$$180^\circ = \pi \text{ rad}$$

$$1 \rightarrow \frac{\pi}{180}$$

$$\frac{\pi}{2} \times \frac{1}{180}$$

Where $\rightarrow k = \frac{1}{2} \frac{g}{u^2}$

Parabolic

23

$y = kx^2$ This equals of Parabolic Path

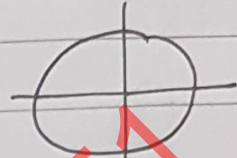
path or Trajectory of this is parabolic

Not

①

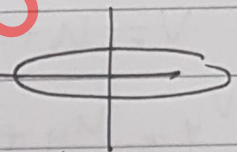
$x + y = 9$

②

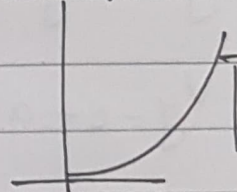


$x^2 + y^2 = 9$

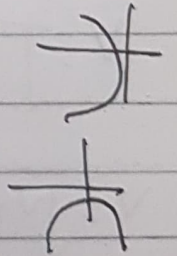
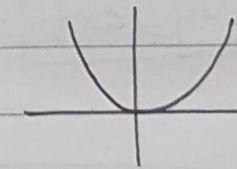
③



④



$y = kx^2$



⑤ Time of flight: This total time for which the projectile is in flight

$S = ut + \frac{1}{2}at^2$

$h = u_y t + \frac{1}{2} a_y t^2$

$h = 0 \times t + \frac{1}{2} \times a \times t^2$

$t^2 = \frac{2 \times h}{a}$

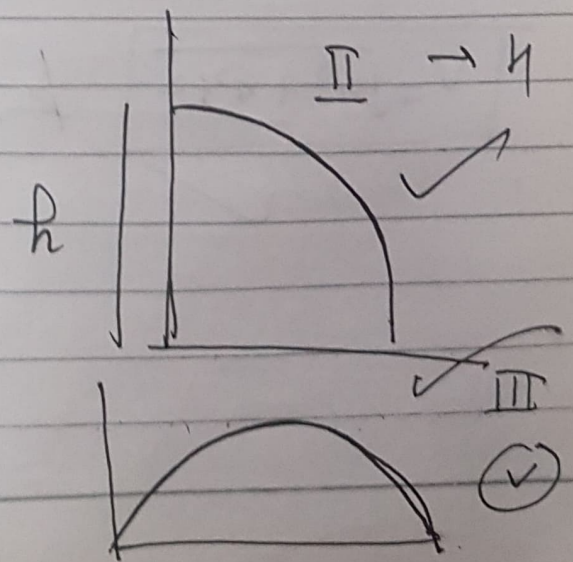
$t = \sqrt{\frac{2h}{a}}$

⑥ Range: This horizontal distance travelled by body

$v = \frac{d}{t} \rightarrow d = v \times t$

$d = u \times \sqrt{\frac{2h}{a}}$

$R = d = u \times \sqrt{\frac{2h}{a}}$



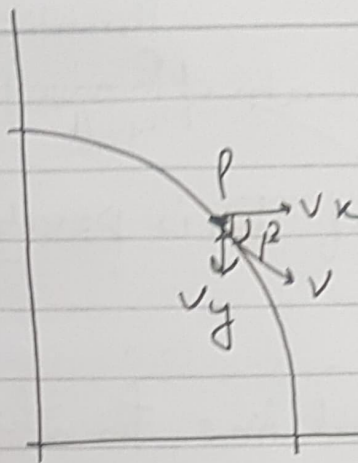
(a) velocity at instat

$$V = u + at$$

$$v_x = u_x + a_x t$$

$$v_x = u + 0 \times t$$

$$v_x = u \quad \text{--- (1)}$$



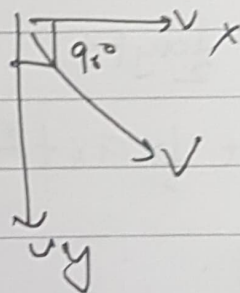
y-axis:

$$V = u + at$$

$$v_y = u_y + a_y t$$

$$v_y = 0 + a t$$

$$v_y = a t \quad \text{--- (2)}$$



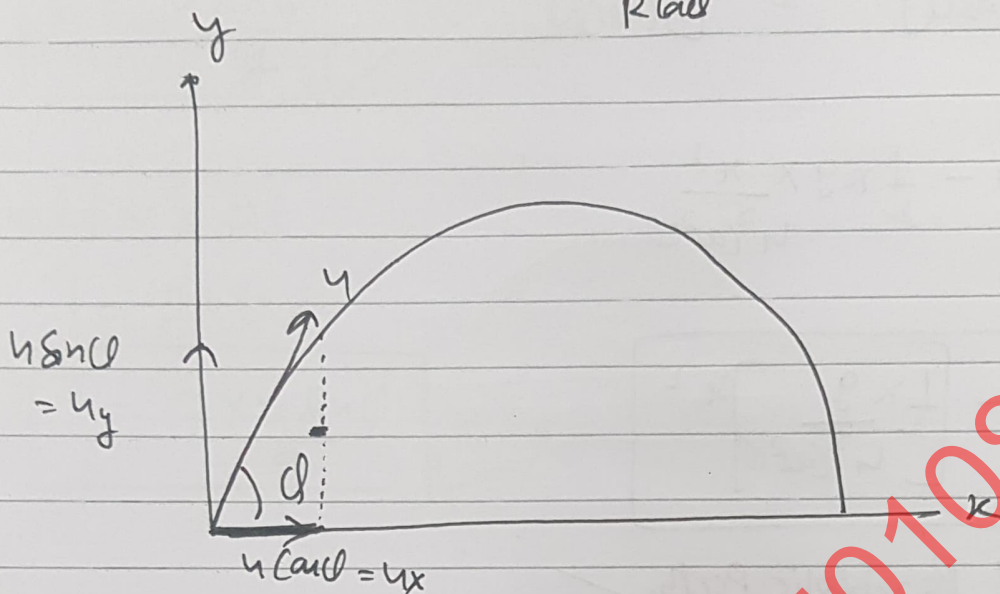
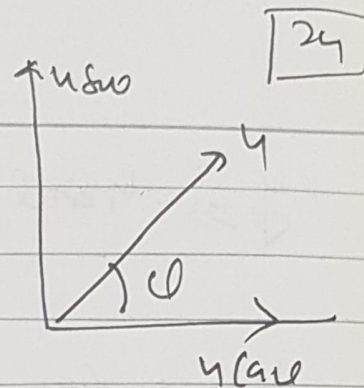
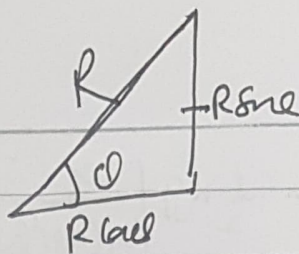
$$V = \sqrt{v_x^2 + v_y^2 + 2v_x v_y \cos 90^\circ}$$

$$\rightarrow V = \sqrt{v_x^2 + v_y^2}$$

$$\rightarrow V = \sqrt{u^2 + (at)^2}$$

$$V = \sqrt{u^2 + a^2 t^2} \quad \checkmark$$

Vertical/Angular Projectile



$$u \begin{cases} u_x = u \cos \theta \\ u_y = u \sin \theta \end{cases}$$

$$a \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

Path: $s = ut + \frac{1}{2}at^2$

X-axis: $X = u_x t + \frac{1}{2}a_x t^2$

$$X = (u \cos \theta) t + \frac{1}{2} \times 0 \times t^2$$

$$X = (u \cos \theta) t$$

$$t = \frac{X}{u \cos \theta}$$

Y-axis: $S = ut + \frac{1}{2}at^2$

$$\rightarrow Y = u_y t + \frac{1}{2}a_y t^2$$

$$\rightarrow Y = (u \sin \theta) t - \frac{1}{2}g t^2$$

$$y = u \sin \theta \left[\frac{x}{u \cos \theta} \right] - \frac{1}{2} \times g \left[\frac{x}{u \cos \theta} \right]^2$$

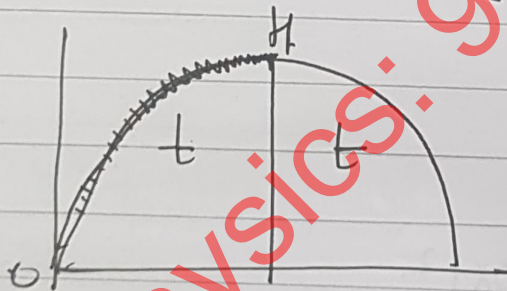
$$y = u \left(\frac{\sin \theta}{\cos \theta} \right) x - \frac{1}{2} \times g \times \frac{x^2}{u^2 \cos^2 \theta}$$

$$y = u \tan \theta x - \left[\frac{1}{2} \frac{g}{u^2 \cos^2 \theta} \right] x^2$$

This is equation of Parabolic path ✓

Time of flight → Total time taken By Body To flight

(T)



$t = \text{Time of ascent}$

$t = \text{Time of descent}$

$$T = t + t = 2t$$

Time taken By Body to reach at Max. height

$$\begin{aligned} \rightarrow V &= u + at \\ \rightarrow V_y &= u_y + a_y t \end{aligned}$$

at Max height height velocity = zero

$$\Rightarrow 0 = u \sin \theta - g t \Rightarrow$$

$$u \sin \theta = gt$$

$$t = \frac{u \sin \theta}{g}$$

$$\text{Total time} = 2t$$

$$T = 2t$$

$$T = \frac{2 \times u \sin \theta}{g}$$

* Maximum height $\rightarrow$ This maximum vertical height attained by object above point of projectile during its flight



$$\rightarrow S = ut + \frac{1}{2}at^2$$

$$\rightarrow H = u_y t + \frac{1}{2}a_y t^2$$

$$t = \frac{u \sin \theta}{g}$$

$$\rightarrow H = (u \sin \theta) \left[\frac{u \sin \theta}{g} \right] - \frac{1}{2} \times g \left[\frac{u \sin \theta}{g} \right]^2$$

$$\rightarrow H = \frac{u^2 \sin^2 \theta}{g} - \frac{1}{2} \times g \times \frac{u^2 \sin^2 \theta}{g^2}$$

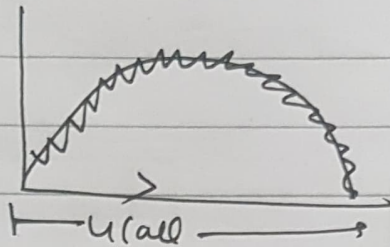
$$H = \frac{u^2 \sin^2 \theta}{g} - \frac{1}{2} \times \frac{u^2 \sin^2 \theta}{g}$$

$$H = \frac{(2u^2 \sin^2 \theta - u^2 \sin^2 \theta)}{2g} = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow H = \frac{u^2 \sin^2 \theta}{g}$$

Range : - This horizontal distance covered by object b/w its points of projection & point of hitting the ~~ground~~ ground.

$$V = \frac{d}{t}$$

$$\underline{d = v \times t}$$



$$d \Rightarrow R = u \times T$$

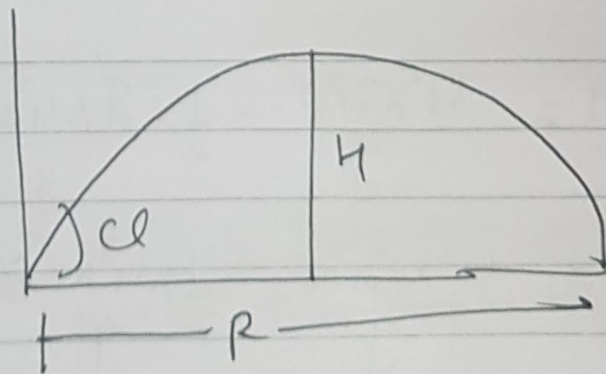
$$R = [u \cos \theta] \times \left[\frac{2u \sin \theta}{g} \right]$$

$$R = \frac{u^2 2 \sin \theta \cos \theta}{g}$$

$$\rightarrow \boxed{R = \frac{u^2 \sin 2\theta}{g}} \quad \Delta$$

Notes

- ① $H = \frac{u^2 \sin^2 \theta}{2g}$
- ② $T = \frac{2u \sin \theta}{g}$
- ③ $R = \frac{u^2 \sin 2\theta}{g}$



NoK

$$P = \frac{u^2 \times \sin 2\theta}{g}$$

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→ 0 to 150

$$P = \frac{u^2 \times \sin(2 \times 45)}{g}$$

$$P = \frac{u^2 \times \sin 90}{g}$$

$$P = \frac{u^2 \times 1}{g}$$

$$P_{\text{max}} = \frac{u^2}{g}$$

$$u = 450$$

$$\rightarrow H = \frac{u^2 \times \sin^2 \theta}{2 \times g}$$

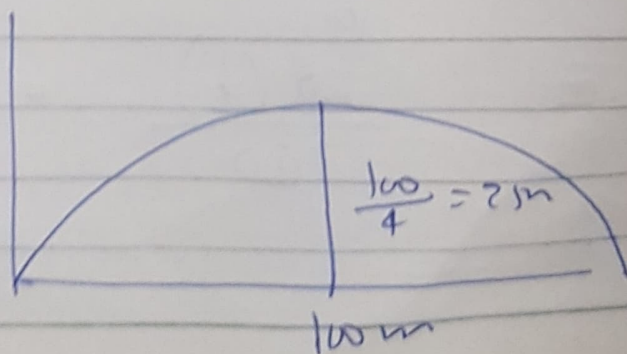
$$\rightarrow H = \frac{u^2 \times \sin^2 45}{2 \times g}$$

$$H = \frac{u^2}{2 \times g} \left(\frac{1}{\sqrt{2}} \right)^2$$

$$H = \frac{u^2}{2g} \times \frac{1}{2} = \frac{1}{4} \left(\frac{u^2}{g} \right)$$

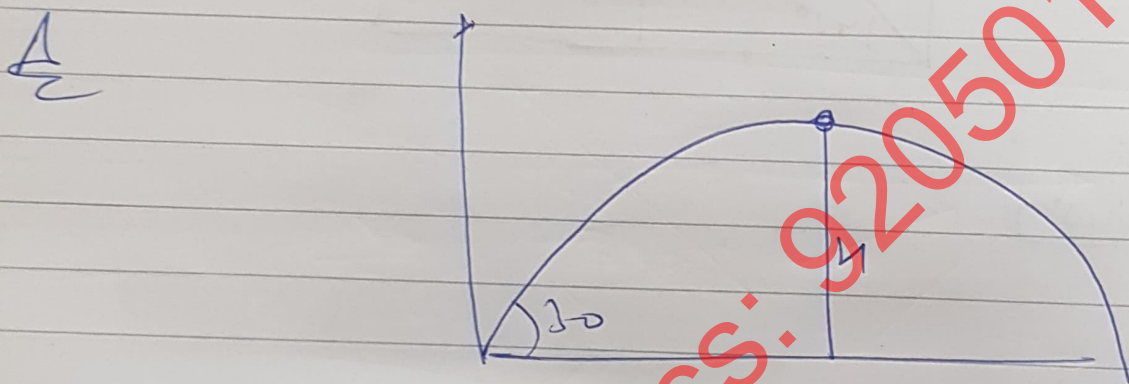
$$H_{\text{max}} = \frac{P_{\text{max}}}{4}$$

eg



- Q. A football player kicks a Ball at angle 30° with horizontal with an initial speed of 20 m/sec . Assuming that the ball travels in vertical plane. Calculate the
- Time at which ball reach the high point
 - Max height Reached
 - horizontal range of Ball
 - The time for which ball in air

$[g = 10 \text{ m/s}^2]$



(a) Total Time of flight: $T = \frac{2u \sin \theta}{g}$

Time taken by Ball to reach max height

$$t = \frac{T}{2} = \frac{u \sin \theta}{g} = \frac{u \sin 30^\circ}{g}$$

$u = 20 \text{ m/s}$ $\theta = 30^\circ$

$$\rightarrow t = \frac{20 \left(\frac{1}{2}\right)}{10} = 1 \text{ sec.}$$

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$$(b) H = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{(20)^2 \left(\frac{1}{2}\right)^2}{2(10)} = \frac{100 \times \frac{1}{4}}{20} = \frac{25}{2} = 12.5 \text{ m}$$

$$(c) \text{ Range of Ball} = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \left(\frac{1}{2}\right)}{10} = \frac{400 \times \frac{1}{2}}{10} = \frac{200}{10} = 20 \text{ m}$$

$$\frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \left(\frac{\sqrt{3}}{2}\right)}{10} = \frac{400 \times \frac{\sqrt{3}}{2}}{10} = \frac{200 \times \sqrt{3}}{10} = 20\sqrt{3} \text{ m}$$

$$(d) \text{ Time of flight} = \frac{2u \sin \theta}{g} = \frac{40 \times \frac{1}{2}}{10} = 2 \text{ sec}$$

Ques find the angle of projection at which horizontal range and maximum height is equal.

Ans

$$\frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{\sin^2 \theta}{2} = 2 \sin \theta \cos \theta$$

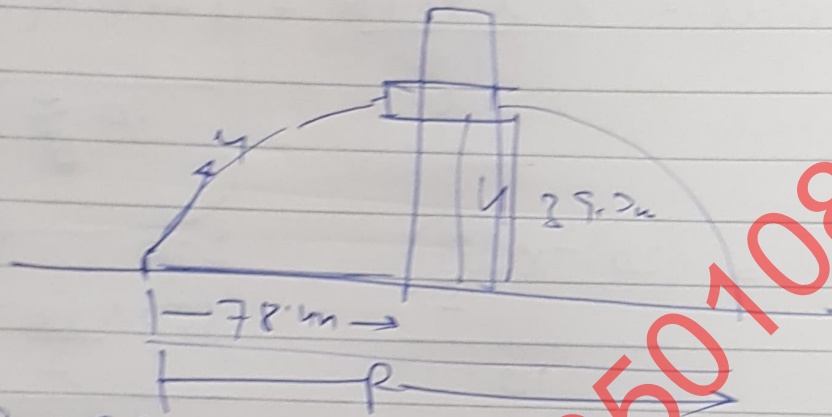
$$\frac{\sin \theta}{2} = 2 \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = 4$$

$$\tan \theta = 4$$

$$\theta = \tan^{-1}(4)$$

Q A Boy stands at 78.4 m from a Building & throws a Ball which just enters a window 39.2 m above the ground. Calculate the velocity of projected Ball



$$\underline{A} \quad H = \frac{u^2 \sin^2 \theta}{2g} = 39.2 \text{ m}$$

$$\text{Horizontal Range} = \frac{u^2 \sin 2\theta}{g} = \frac{78.4 \times 2}{g}$$

$$\rightarrow \frac{\frac{u^2 \sin^2 \theta}{2g}}{\frac{u^2 \sin 2\theta}{g}} = \frac{39.2}{2 \times 78.4}$$

$$\rightarrow \frac{\frac{u^2 \cancel{\sin^2 \theta}}{2g} \times \frac{g}{\cancel{u^2} \times 2 \sin 2\theta}}{1 \times 78.4} = \frac{39.2}{1 \times 78.4}$$

$$\rightarrow \frac{\cancel{u^2} \times \cancel{g}}{2 \times \cancel{u^2} \times \cancel{g} \times 2 \sin 2\theta} = \frac{39.2}{78.4 \times 2}$$

$$\therefore \sin 2\theta = \frac{39.2 \times 2}{78.4} = \frac{78.4}{78.4} \rightarrow \sin 2\theta = 1$$

$$\boxed{2\theta = 90^\circ}$$

$$\rightarrow \frac{4^2 \times 8^2 \times 4}{2g} = 38.2$$

$$\rightarrow \frac{4^2 \times 1}{2g \times 1^2} = 38.2$$

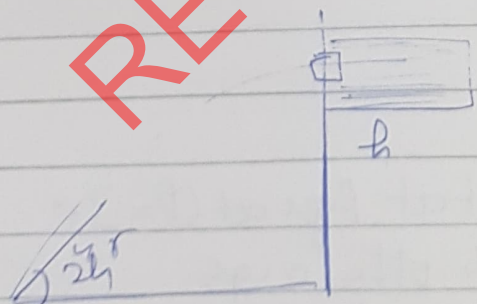
$$\rightarrow \frac{4^2 \times 1}{g \times 2 \times 2} = 38.2$$

$$\rightarrow \frac{4^2}{4 \times g} = 38.2$$

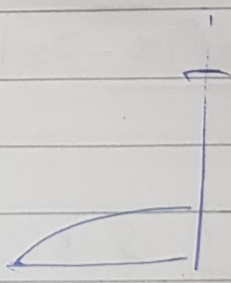
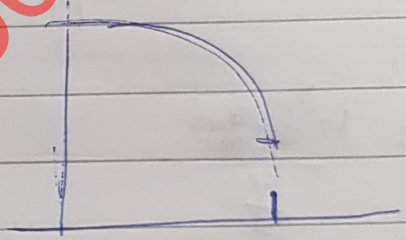
$$\rightarrow 4^2 = h \times 4 \times g$$

$$\rightarrow 4 = \sqrt{h \times 4 \times g}$$

$$\rightarrow 4 = \sqrt{38.2 \times 4 \times 10}$$

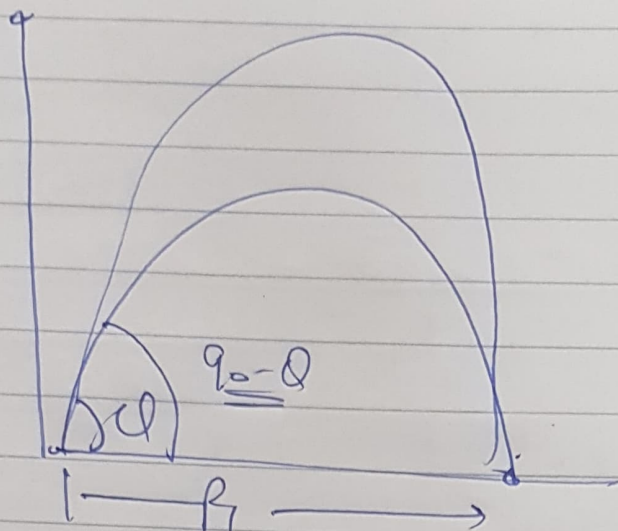


$$4 = \sqrt{2 \times g \times h}$$



RE Physics: 9205010851

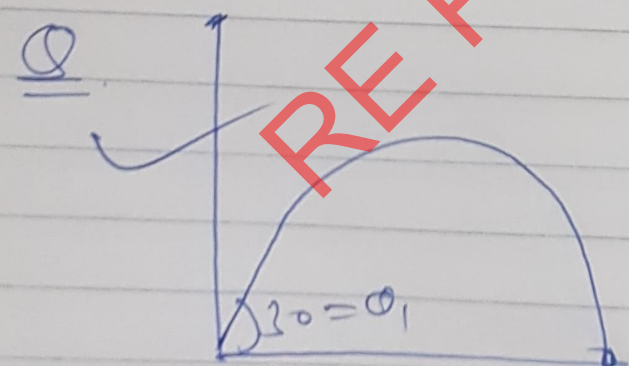
* Two angle of projection, for same horizontal Range



$$\rightarrow R_1 = \frac{u^2 \sin 2\theta}{g}$$

$$\rightarrow R_2 = \frac{u^2 \times \sin(180 - 2\theta)}{g} = \frac{u^2 \times \sin 2\theta}{g} = \frac{u^2 \sin 2(90 - \theta)}{g}$$

$\Rightarrow R_1 = R_2$ If θ is change $(180 - \theta)$



a Ball hit at Range at $\theta = 30^\circ$
By which other angle
Should be kick for
Same Range?

$$\theta = 90 - \theta$$

$$\theta = 90 - 30$$

$$\theta = 60^\circ$$