Chapter no 1: Electrostatic

Part-1 Electric Forces and Field

1- Change on Body is in integral form is called Quantization of charge

$q = \pm ne$ where: n = 1, 2, 3, _ _ _ _ _

Let ‘e’ is the charge on body

Proton = +q = +e =1.6×10^{-19}C

Electron = −q = –e = -1.6×10^{-19}C

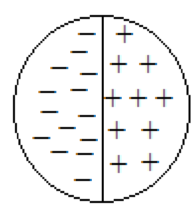
Conduction and Induction

Property of charges

q_1	q_2	Property
+	+	Repel
+	–	Attract
–	+	Attract
–	–	Repel

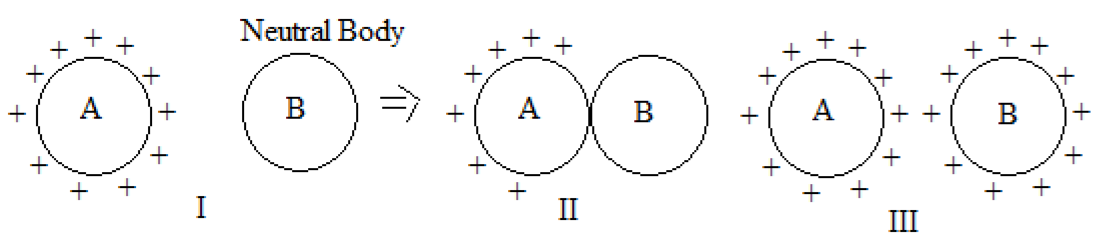
Like charge always repel
Unlike charges always attract

Every Body is Neutral body mean equal number of +ve charge = equal number of negative charge
Net charge = nq − nq = 0
But any Neutral body become positive charge body or negative charge body.



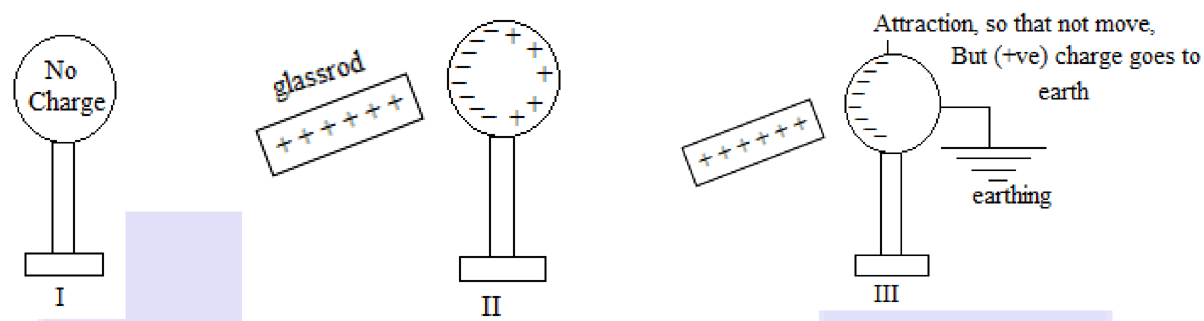
Conduction

A positive charge body connected to neutral body than neutral body converted into positive charge body.

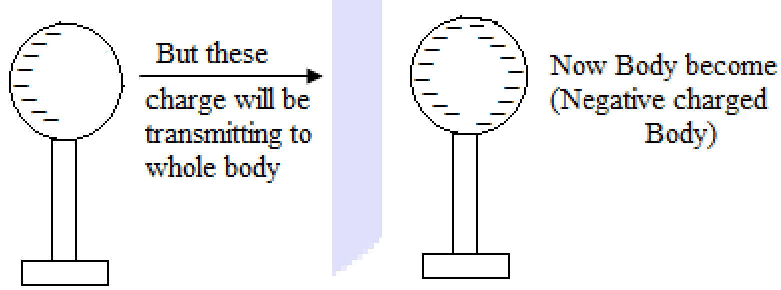


Induction

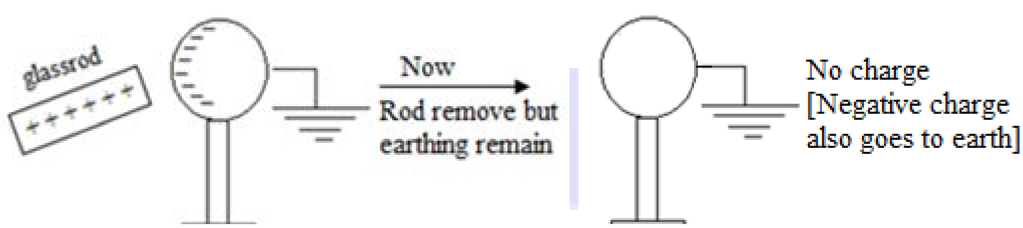
Phenomena of charging an uncharged conducting body, by bringing a charged body near it, without making a direct contact between the two bodies called "charging by induction".



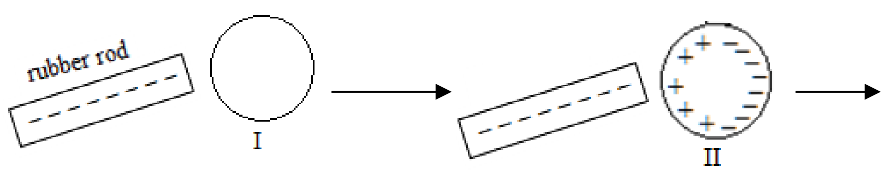
Now, Rod & Earthing → Remove

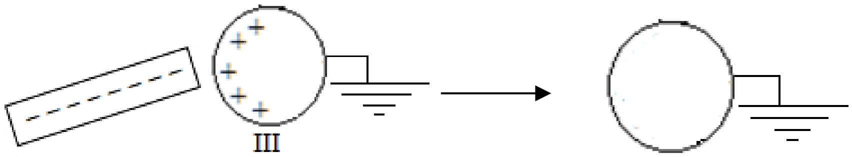


Eg.

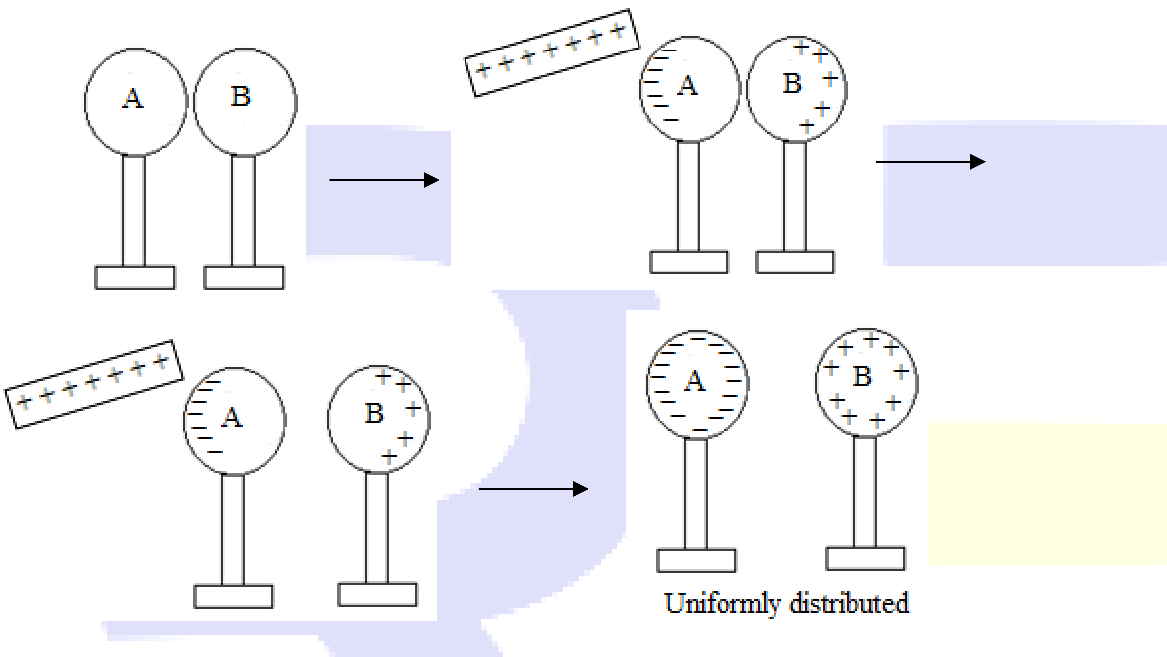


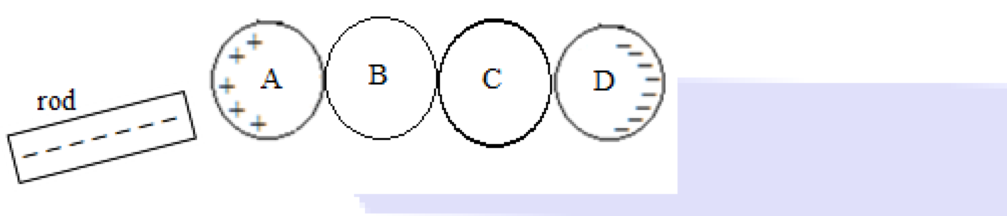
Eg.





Note





2- Rate of change of charge is called "current"

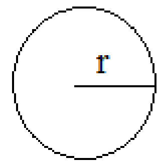
$I = \frac{dq}{dt}$, $q = ne$

$I = \frac{ne}{t} = ne \times \nu$ where $T = \frac{1}{\nu}$

T is Time period and ν is frequency

3- If 'e' is electron moves in circular path with velocity (v) and radius (r)

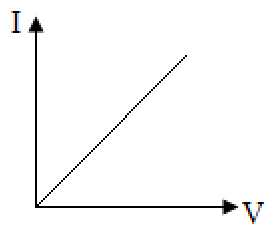
$V = \frac{d}{T} = \frac{2\pi r}{T}$

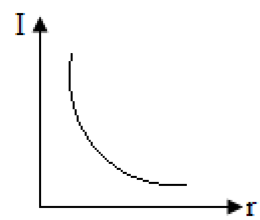


$T = \frac{2\pi r}{V}$

Then current is

$I = \frac{q}{T} = \frac{ne}{T} = \frac{ne}{2\pi r/V} = \frac{ne \times V}{2\pi r}$





Current: $I = \frac{ne \times V}{2\pi r}$

Potential: $V = I \times R = \left[\frac{ne \times V}{2\pi r}\right] \times R$

Where R is resistance

4) Coulomb's force

$f = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2}$

where: $\frac{1}{4\pi\varepsilon_0} = K = 9 \times 10^9\ Nm^2c^{-2}$ and $\varepsilon_0 = 8.85 \times 10^{-12}\ N^{-1}c^2m^{-2}$

Unit of $\varepsilon_0 = N^{-1}c^2m^{-2}$

Dimension of $\varepsilon_0 = M^{-1}L^{-3}T^2C^2$

Unit of $K = Nm^2c^{-2}$

Dimension of $K = ML^3T^{-4}A^{-2}$

ε_0 = Electric Permittivity of free space

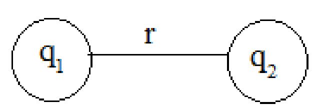
ε = Electric Permittivity of medium

ε_r = Relative electrical Permittivity of Medium with space

$\varepsilon = \varepsilon_0 \times \varepsilon_r$

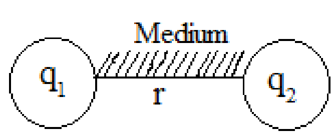
Note: f_1 is a force between two charge and f_2 is new force when free air is replaced by another medium.

$f_1 = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2}$



$f_2 = \frac{1}{4\pi\varepsilon} \frac{|q_1||q_2|}{r^2}$

$\varepsilon = \varepsilon_r \times \varepsilon_0$



$f_2 = \frac{1|q_1||q_2|}{4\pi\varepsilon_0 \times \varepsilon_r r^2}$

New force after replace air/ free space by medium.

$f_2 = \frac{f_1}{\varepsilon_r}$

Note:

* ε_r is 81 for water

* ε_r is ∞ for metal

* $\varepsilon_r \geq 1$

5- Resultant of Coulomb forces

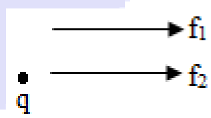
Let there are two forces then resultant force will depend upon the direction of forces.

Case 1: When both force in same direction

Resultant force will be

$f_R = f_1 + f_2$

Example: 1



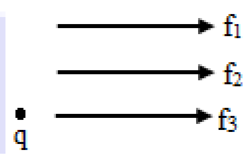
$f_1 = 2N, f_2 = 3N$

$f = f_1 + f_2 = 5N$

Example: 2

$f_1 = 2N, f_2 = 4N, f_3 = 5N$

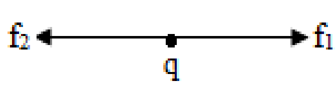
$f = f_1 + f_2 + f_3 = 11N$



Case 2: When both force in opposite direction

Resultant force will be

$f_N = f_1 - f_2$



Example: 1

$f_1 = 12N, f_2 = 3N$

$f_N = f_1 - f_2 = 12 - 3 = 6N$

Example: 2

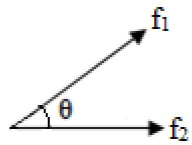
$f_1 = 10N, f_2 = 20N, f_3 = 5N$

$f = f_1 + f_2 - f_3 = 25N$

Case 3: When angle (θ) is between two forces

Resultant force will be

$$f = \sqrt{f_1^{\,2} + f_2^{\,2} + 2f_1 f_2 \cos\theta}$$

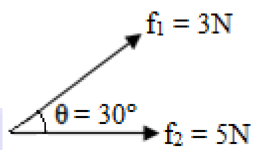


Example 1:

$$f = \sqrt{f_1^{\,2} + f_2^{\,2} + 2f_1 f_2 \cos\theta}$$

$$\sqrt{(3)^2 + (5)^2 + 2 \times 3 \times 5 \times \cos 30}$$

$$= \sqrt{9 + 25 + 30\frac{\sqrt{3}}{2}}$$

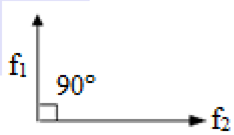


- **Special Case**

Case 1: When θ is 90°

$$f = \sqrt{f_1^{\,2} + f_2^{\,2} + 2f_1 f_2 \cos 90}$$

$$f = \sqrt{f_1^{\,2} + f_2^{\,2}}$$

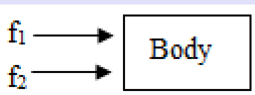


Case 2: When θ = 0°

$$f = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2 \cos 0}$$

$$f = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2}$$

$$f = \sqrt{(f_1 + f_2)^2}$$



$f = f_1 + f_2$ (Maximum force or greatest force)

Case 3: When θ = 180°

$$f = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2 \cos 180}$$

$$f = \sqrt{f_1^2 + f_2^2 - 2f_1 f_2}$$

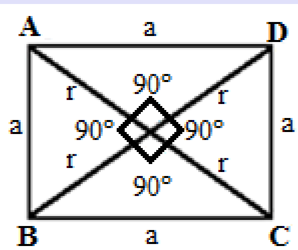
f = $\sqrt{(f_1 - f_2)^2}$

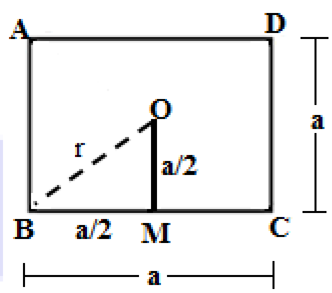
f = $f_1 - f_2$ (Minimum force or least fore

Important Results:

Angle	Resultant Force
θ = 0°	$f_N = \sqrt{f^2 + f^2 + 2f^2} = 2\sqrt{2}$
θ = 30°	$f_N = \sqrt{f^2 + f^2 + 2f^2 \times \frac{\sqrt{3}}{2}} = f\sqrt{(2 + \sqrt{3})}$
θ = 45°	$f_N = \sqrt{f^2 + f^2 + \frac{2f^2}{\sqrt{2}}} = f\sqrt{\left(2 + \frac{2}{\sqrt{2}}\right)} = f\sqrt{(2 + \sqrt{2})}$
θ = 60°	$f_N = \sqrt{f^2 + f^2 + 2 \times f^2 \times \frac{1}{2}} = f\sqrt{3}$
θ = 90°	$f_N = \sqrt{f^2 + f^2 + 2f.f\cos 90} = \sqrt{f^2 + f^2} = f\sqrt{2}$
θ = 180°	$f_N = \sqrt{f^2 + f^2 + 2f.f\cos 180} = \sqrt{2f^2 - 2f^2} = 0$

ABCD is a square, AB = BC = CD = AD = A,





$$r = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}$$

$$r = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \sqrt{\frac{2a^2}{4}} = \frac{a}{\sqrt{2}}$$

AO = BO = CO = DO = r = $\frac{a}{\sqrt{2}}$

Force on O by A = $f_A = \frac{k|q_A||q_0|}{(AO)^2}$

Force on O by B = $f_B = \frac{k|q_B||q_0|}{(OB)^2}$

Force on O by C = $f_C = \frac{k|q_C||q_0|}{(OC)^2}$

Force on O by D = $f_D = \frac{k|q_D||q_0|}{(OD)^2}$

Note:

1) –q charge is always is mode, so it will become positive in all forces.

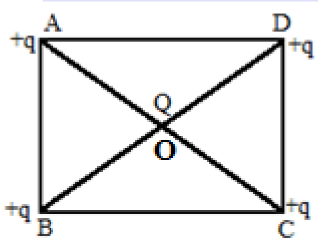
$f_A = f_B = f_C = f_D = f$

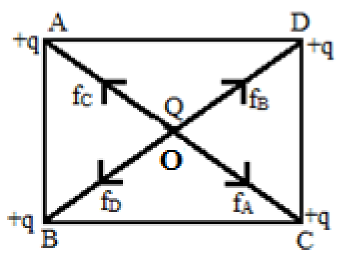
2) Direction of force from +q is outside ⟶

3) Direction of force from –q is inward ⟵

Direction of coulomb forces

Ques.1 Find the force on 'O'.



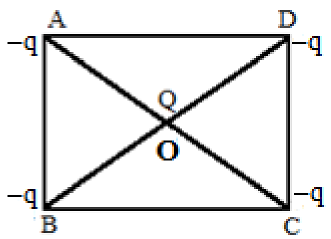


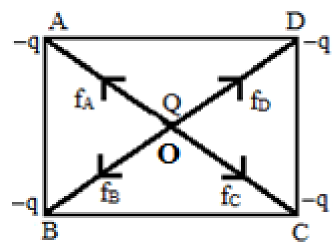
Ans. f_A and f_C are equal and opposite in direction so they cancel out each other.

Similarly f_D and f_B are equal and opposite in direction so they cancel out each other.

So net force, $F_N = 0$

Ques.2 Find the force on 'O'.



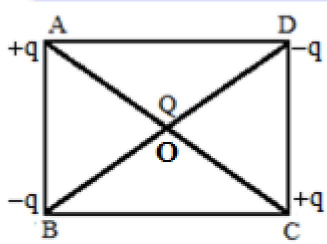


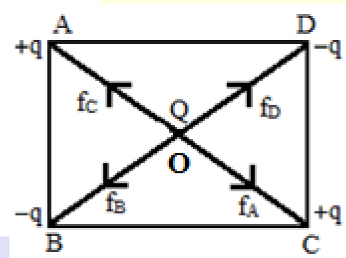
Ans. f_A and f_C are equal and opposite in direction so they cancel out each other.

Similarly f_D and f_B are equal and opposite in direction so they cancel out each other.

So net force, $F_N = 0$

Ques.3 Find the force on 'O'.



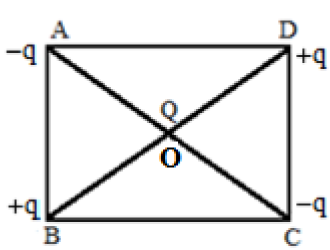


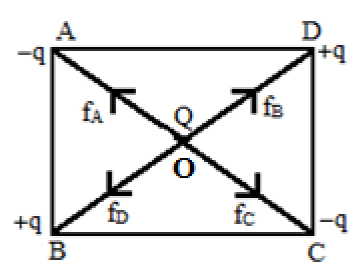
Ans. f_A and f_C are equal and opposite in direction so they cancel out each other.

Similarly f_D and f_B are equal and opposite in direction so they cancel out each other.

So net force, $F_N = 0$

Ques.4 Find the force on 'O'.



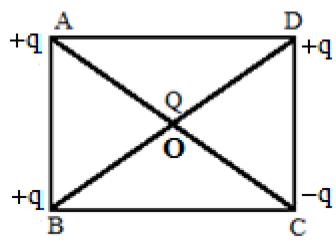


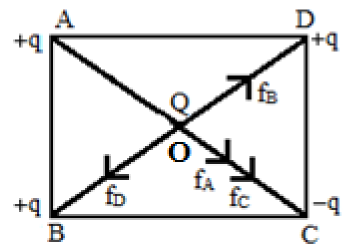
Ans. f_A and f_C are equal and opposite in direction so they cancel out each other.

Similarly f_D and f_B are equal and opposite in direction so they cancel out each other.

So net force, $F_N = 0$

Ques.5 Find the force on 'O'.

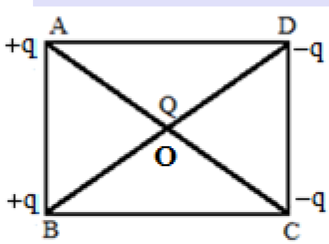


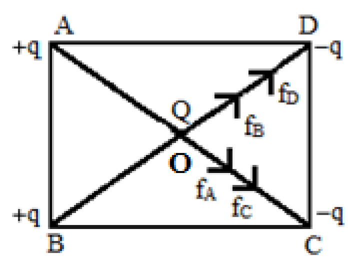


Ans. f_B and f_D are equal and opposite force and f_A and f_C are in same direction

So net force, $F_N = f_A + f_C = f + f = 2f = 2 \times \frac{kQq}{r^2} = \frac{2kQq}{(a/\sqrt{2})^2} = \frac{4kQq}{r^2}$

Ques.6 Find the force on 'O'.





Ans. Let $f_A = f_B = f_C = f_D = f$

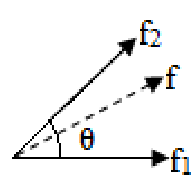
f_B and f_D are in same direction, so

$f_1 = f_A + f_C = f + f = 2f$

f_A and f_C are in same direction

$f_2 = f_D + f_B = f + f = 2f$

$f = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos\theta}$



$f_N = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos 90}$

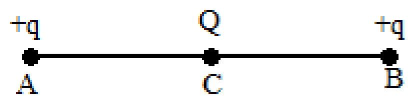
$$f_N = \sqrt{(2f)^2 + (2f)^2}$$

$$f_N = \sqrt{4f^2 + 4f^2}$$

$$f_N = 2\sqrt{2}f = 2\sqrt{2}\,\frac{kQq}{r^2} = 2\sqrt{2}\,\frac{kQq}{(a/\sqrt{2})^2} = \frac{4\sqrt{2}kQq}{a^2}$$

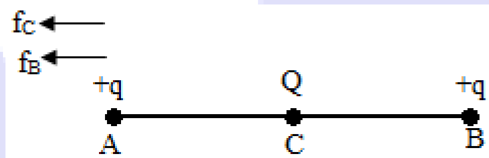
Case of equilibrium

(I)



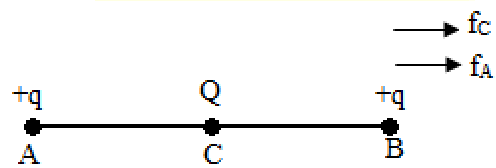
1) Force on 'A'

$f = -f_B - f_C$



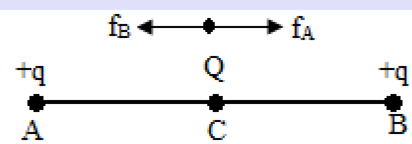
2) Force on 'B'

$f = f_A + f_C$

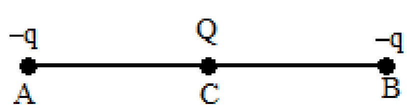


3) Force on 'C'

$f = f_A - f_B$

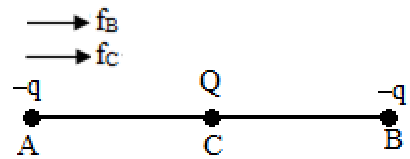


(II)



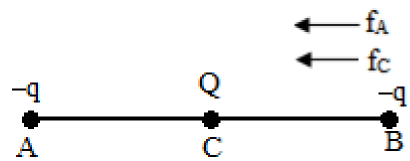
1) Force on 'A'

$f = f_B + f_C$



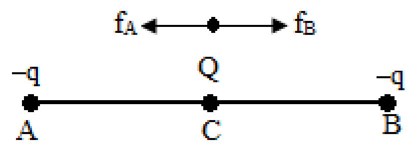
2) Force on 'B'

$f = -f_A - f_C$



3) Force on 'C'

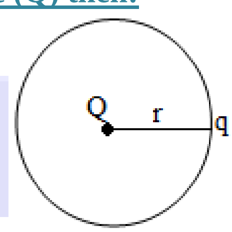
$f = f_B - f_A$



6- A charge (q) is moving in circular path around the charge (Q) then:

Centripetal force on charge (Q) = $f_C = \frac{mv^2}{r}$

Force between q and Q = $f = \frac{kq \times Q}{r^2}$



For uniform circular motion, both force are equal

$f_C = f$

Radius

$$\frac{k\,q\,Q}{r^2} = \frac{mv^2}{r}$$

$$r = \frac{k\,q\,Q}{v^2}$$

Velocity (v)

$$\frac{k\,q\,Q}{mr} = v^2$$

$$v = \sqrt{\frac{k\,q\,Q}{mr}}$$

Angular velocity (ω)

$$v = r\omega$$

$$\omega = \frac{v}{r}$$

$$\omega = \frac{\sqrt{\frac{k\,q\,Q}{mr}}}{r} = \sqrt{\frac{k\,q\,Q}{mr.r^2}} \Rightarrow \omega = \sqrt{\frac{k\,q\,Q}{mr^3}}$$

Time period (T)

$$\omega = 2\pi\nu = 2\pi \times \frac{1}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{mr^3}{k\,q\,Q}}$$

Frequency (ν)

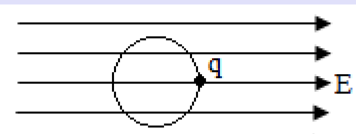
$$\omega = 2\pi\nu$$

$$\nu = \frac{\omega}{2\pi}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k\,q\,Q}{mr^3}}$$

7-A charge particle (q) moving in circular path in electric field

Force on charge particle (q) in electric field



$$f = q\,E$$

Centripetal force on charge particle

$$f_C = \frac{mv^2}{r}$$

For uniform circular motion, both force are equal

$$f = f_C$$

$$q\,E = \frac{mv^2}{r}$$

radius (r)

$$r = \frac{mv^2}{q\,E}$$

Velocity (v)

$$v = \sqrt{\frac{q\,E\,r}{m}}$$

Angular velocity (ω)

$$V = r \times \omega$$

$$\omega = \frac{v}{r} = \sqrt{\frac{q\,E\,r}{mr^2}} = \sqrt{\frac{q\,E}{mr}}$$

$$\omega = \sqrt{\frac{q\,E}{mr}}$$

Time period (T)

$$\omega = 2\pi\nu = 2\pi \times \frac{1}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{mr}{q \times E}}$$

Frequency (ω)

$$\omega = 2\pi\nu$$

$$\nu = \frac{\omega}{2\pi}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{qE}{mr}}$$

8- A field around a charge due to a given charge as the field that permeates the space around the charge, in which electrostatic force of attraction or repulsion due to charge can be experienced by any other charge.

$$E = \lim_{q_0 \to 0} \frac{f}{q_0}$$

$$E = \frac{\frac{1}{4\pi\varepsilon_0}\frac{|Q||q_0|}{r^2}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2}$$

Unit of Electric field

$f = qE$

$E = \frac{f}{q} = \frac{\text{Newton}}{\text{Coulomb}} = \frac{N}{C} = NC^{-1}$

Dimension of Electric field

$E = \frac{f}{q} = \frac{MLT^{-2}}{AT} = MLT^{-3}A$

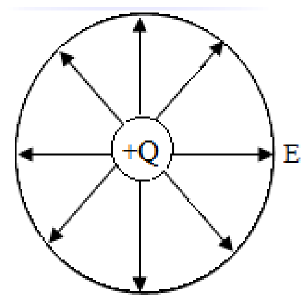
Electric field is a vector quantity

When, (q_0) is test charge, which is used to find electric field of other change (Q)

Property of Test charge (q_0)

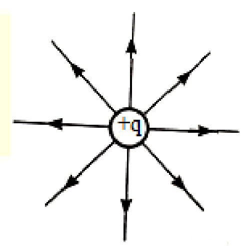
1) Electric field due to Test charge (q_0) is negligible

2) Magnitude of Test charge (q_0) is one (1).

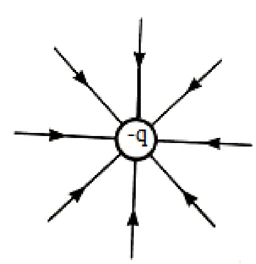


Electric field lines for different charge systems

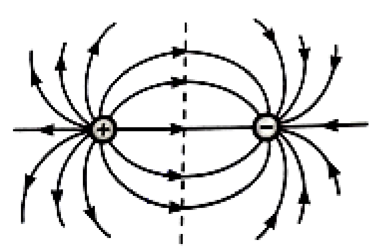
1) Electric field lines of a positive point charge



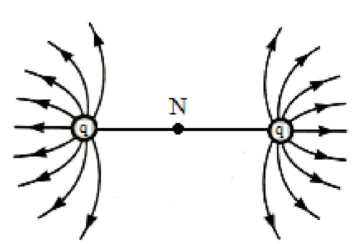
2) Electric field lines of a negative point charge



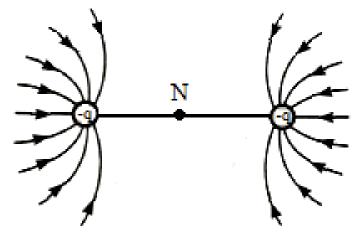
3) Electric field lines of an electrical dipole



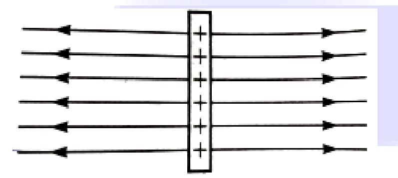
4) Electric field lines of two positive charge



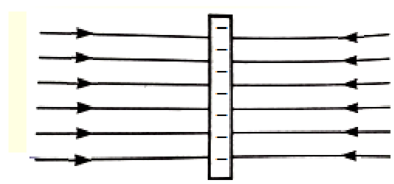
5) Electric field lines of two negative charge



6) Electric field lines of a positive charged plane conductor



7) Electric field lines of a negative charged plane conductor



Note:

Neutral point – The point where no electric field lines passes

Condition of neutral point

1) Both charge should be positive

2) Both charge should be negative

9-Path of charge particle (q) when it move in electric field (E)

Electric field $E = \frac{kq}{r^2}$

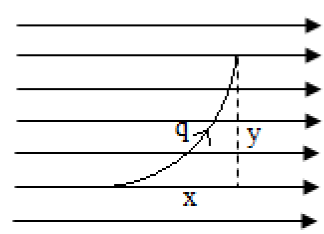
For charge (q)

$E = \frac{f}{q} \Rightarrow f = qE$

Force on charge (q) when it move in electric field (E)

f = qE

Path of charge particle (q) when it move in electric field (E)



Force on charge particle in electric field f = qE

When charge particle move in electric field with acceleration (a) then force will be

$f = ma$

$ma = qE$

$a = \frac{qE}{m}$

Then equation of path covered in y – axis is

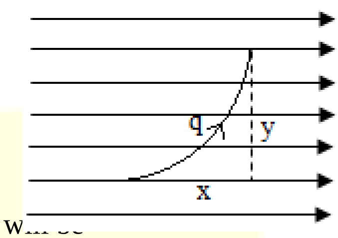
$y = \frac{1}{2}\left(\frac{qE}{m}\right)\frac{x^2}{v^2}$

$y = kx^2$, when $k = \frac{1}{2}\frac{qE}{mv^2}$

The path or trajectory of charge particle is parabolic

10-Velocity and Distance travelled by charge particle in electric field

Force on charge (q) when it move in electric field (E)



$f = qE$

Path of charge particle (q) when it move in electric field (E)

Force on charge particle in electric field $f = qE$

When charge particle move in electric field with acceleration (a) then force will be

$f = ma$

$ma = qE$

$a = \frac{qE}{m}$

initial velocity is zero for charge particle

$u = 0$

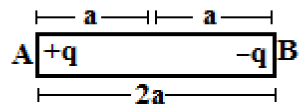
$v = u + at \quad \Rightarrow \quad v = \frac{qE}{m}.t$

$S = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad S = \frac{1}{2}\frac{qE}{m}.t^2$

$v^2 = u^2 + 2as \quad \Rightarrow \quad v^2 = 2\frac{qE}{m}s$

$v = \sqrt{2\frac{qE}{m}s}$

11- Electric dipole: A pair of equal & opposite charge +q and –q separated by a small distance is called "electric dipole"



Electrical Dipole Moment = Either charge × Distance between them

$P = |q| \times 2a$

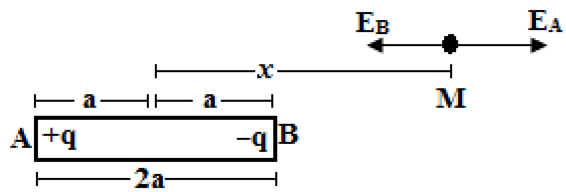
Unit P = Coulomb × metre = C × m

Dimensional P = [AT][L] = [ATL]

Direction of electric dipole is (–ve) to (+ve)

Direction of electric field is (+ve) to (–ve)

12- Electric field due to dipole at axial point



$$E_a = \frac{2k(q2a)x}{(x^2-r^2)^2} = \frac{2\,k\,P\,x}{(x^2-r^2)^2}$$

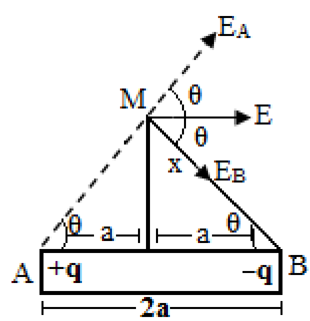
$P = q\,2a$

Special Case

$a^2 <<< r^2$ so a^2 neglect

$$E_a = \frac{2kPx}{(x^2-0^2)^2} = \frac{2kPx}{x^4} = \frac{2kP}{x^3}$$

13- Electric field due to dipole at equatorial point



$$E_e = \frac{k\,q\,2a}{(x^2+a^2)^{3/2}} = \frac{k\,P}{(x^2+a^2)^{3/2}}$$

$P = q\,2a$

Special Case

$a^2 <<< r^2$ so a^2 neglect

$$E_e = \frac{k\,q\,2a}{(x^2+0^2)^{3/2}} = \frac{kP}{x^3}$$

Relation between an electric field at axial point (E_a) and equatorial point (E_e)

electric field at axial point = 2 × electric field at equatorial point

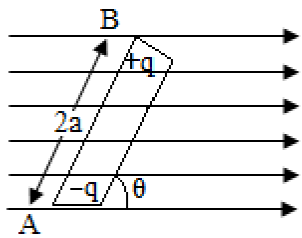
$E_a = 2 \times E_e$

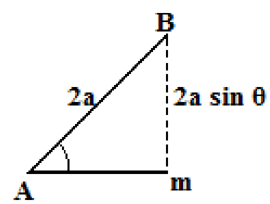
14-Torque on a dipole in a uniform electric field

An electric dipole is placed in electric field at angle (θ). It will experience a Torque (t)

$\tau = PE \sin\theta$ $\quad\quad$ ($\because P = q \times 2s$)

$\vec{\tau} = \vec{P} \times \vec{E}$ $\quad\quad$ ($\vec{A} \times \vec{B} = AB \sin\theta$)





Work done in rotating the electrical dipole through an angle θ in electric field (E)

Let electrical dipole rotate from θ_1 to θ_2

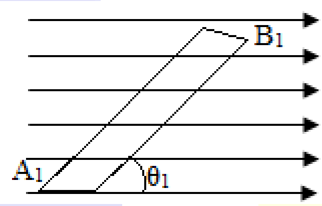
Then work done will be

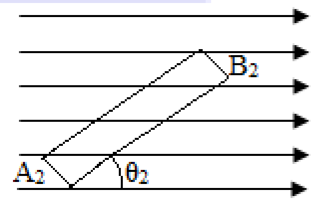
$w = -PE (\cos\theta_2 - \cos\theta_1)$

θ_1 = initial angle

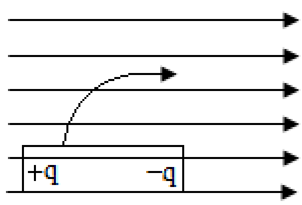
θ_2 = final angle

work done by electric dipole

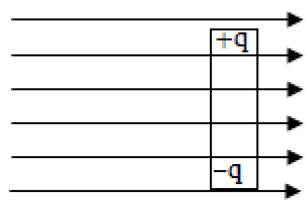




1) Stable condition



$\theta_1 = 90°$

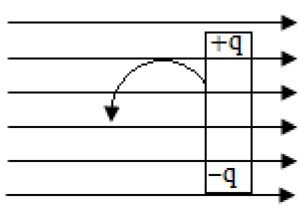


$\theta_2 = 0°$

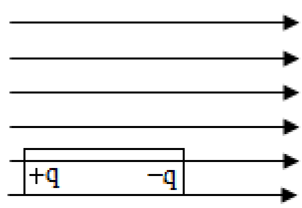
$w = -PE (\cos\theta_2 - \cos\theta_1)$

$w = -PE (\cos 0° - \cos 90°) = -PE (1 - 0) = -PE$

2) Unstable condition



$\theta_1 = 90°$



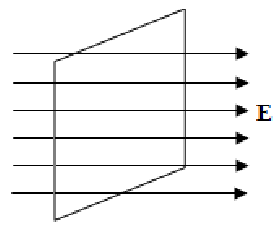
$\theta_2 = 180°$

$w = -PE(\cos\theta_2 - \cos\theta_1)$

$w = -PE(\cos 180° - \cos 90°) = -PE(-1 - 0) = PE$

15- Electric flux (ϕ) is measure of total number of electric lines of the force passing normally through that area.

$\phi = \vec{E} \cdot \vec{S} = E.\, S \cos\theta$ $\qquad (\vec{A} \cdot \vec{B} = AB\cos\theta)$

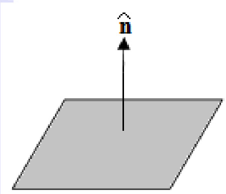


θ is angle between Electric field (E) & Area vector ($\hat{n}$)

Electric flux is a scalar quantity

Every area has area vector ($\hat{n}$)

$\hat{n}$ is always perpendicular to the surface

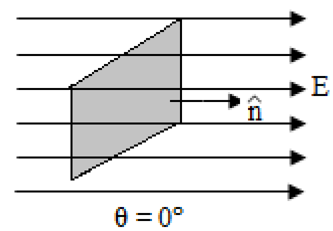


Case 1: Maximum flux

Angle between E & $\hat{n}$,

$\theta = 0°$

$\phi = \vec{E} \cdot \vec{S} = E.S\cos\theta = E.S\cos 0 = E.S$

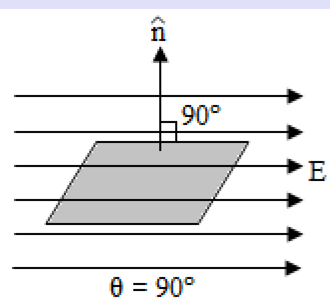


Case 2: Minimum flux

Angle between E & $\hat{n}$,

$\theta = 90°$

$\phi = \vec{E} \cdot \vec{S} = E.S\cos\theta = E.S\cos 90 = 0$



Note:

1) Normal form of electrical flux

$\phi = \vec{E} \cdot \vec{S} = E.S\cos\theta$

2) Integral form of electrical flux

Let dS is a small surface area and $\oint_S$ is integral of close surface area

$\phi = \oint_S \vec{E}.d\vec{S} = \oint_S E.dS\cos\theta$

3) Electric Flux density $\frac{\phi}{A} = E$

4) Charge density

λ is Linear charge density, $\lambda = q/l$

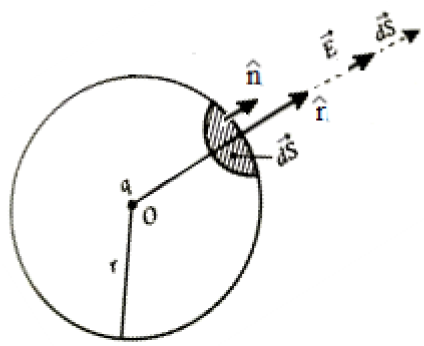
σ is Surface charge density, $\sigma = q/A$

ρ is Volume charge density, $\rho = q/V$

l is length, A is area and V is volume

16- Gauss theorem states that the total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface.

$\phi = \oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$

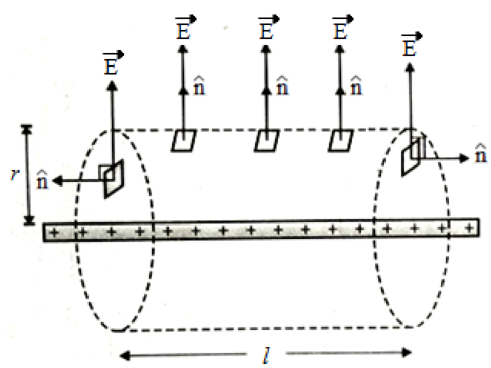


Application of Gauss Theorem

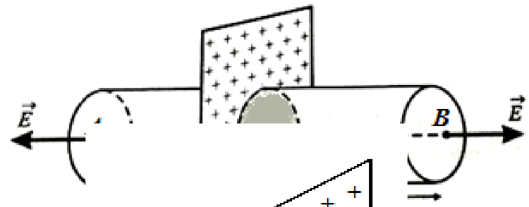
1) Electric field due to an infinitely long straight charged wire.

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Where, λ is Linear charge density, $\lambda = q/l$



2) Electric field due to a uniformly charged infinite plane sheet.



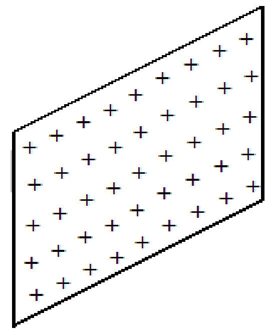
$$E = \frac{\sigma}{2\varepsilon_0}$$

σ is Surface charge density, σ = q/A

Special Case:

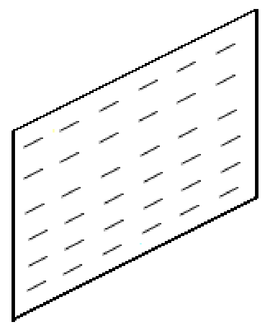
1) Thin sheet

Electric field due to positive thin sheet.



$$E = \frac{\sigma}{2\varepsilon_0}$$

Electric field due to negative thin sheet.



$$E = -\frac{\sigma}{2\varepsilon_0}$$

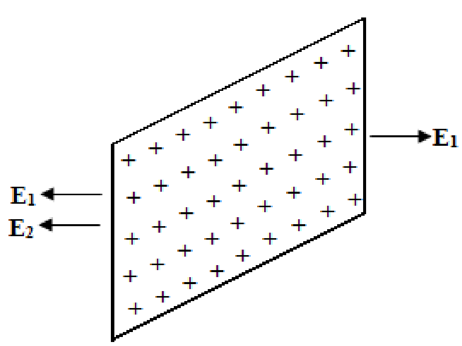
σ is negative for negative sheet

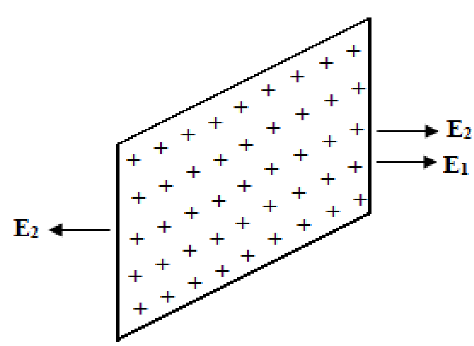
Combination:

Case – 1: Thin sheet when both are positive

$$E_1 = \frac{\sigma}{2\,\varepsilon_0}$$

$$E_2 = \frac{\sigma}{2\,\varepsilon_0}$$



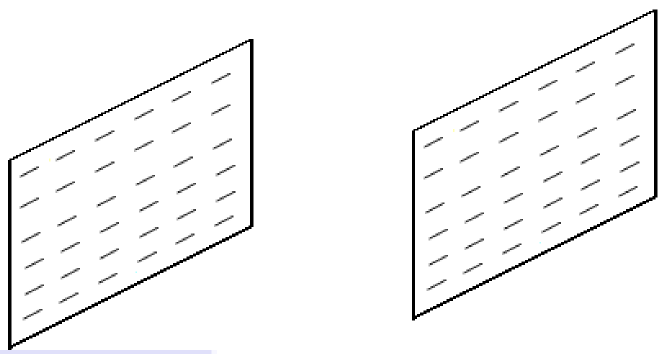


I Region	II Region	III Region
$E = -E_1 - E_2$ $E = -\frac{\sigma}{2\,\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$ $E = -\frac{2\sigma}{2\,\varepsilon_0}$ $E = -\frac{\sigma}{\varepsilon_0}$	$E = E_1 - E_2$ $E = \frac{\sigma}{2\,\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$ $E = 0$	$E = E_1 + E_2$ $E = \frac{\sigma}{2\,\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$ $E = \frac{2\,\sigma}{2\,\varepsilon_0}$ $E = \frac{\sigma}{\varepsilon_0}$

Case – 2: Thin sheet when both are negative

$$E_1 = -\frac{\sigma}{2\,\varepsilon_0}$$

$$E_2 = -\frac{\sigma}{2\,\varepsilon_0}$$

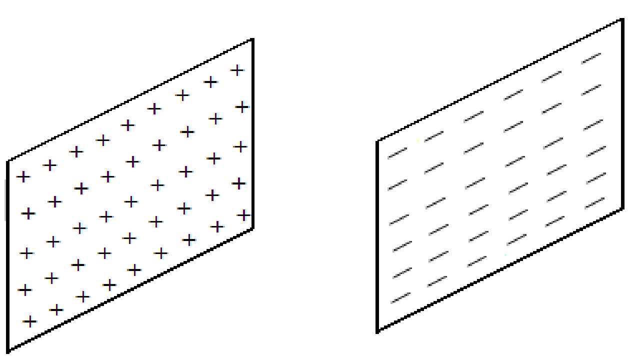


I Region	II Region	III Region
$E = -E_1 - E_2$ $E = -\left(-\frac{\sigma}{2\,\varepsilon_0}\right) - \left(-\frac{\sigma}{2\varepsilon_0}\right)$ $E = \frac{2\sigma}{2\,\varepsilon_0}$ $E = \frac{\sigma}{\varepsilon_0}$	$E = E_1 - E_2$ $E = -\frac{\sigma}{2\,\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right)$ $E = 0$	$E = E_1 + E_2$ $E = -\frac{\sigma}{2\,\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$ $E = \frac{-2\sigma}{2\,\varepsilon_0}$ $E = -\frac{\sigma}{\varepsilon_0}$

Case – 3: When one thin sheet is positive and other is negative

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

$$E_2 = -\frac{\sigma}{2\varepsilon_0}$$

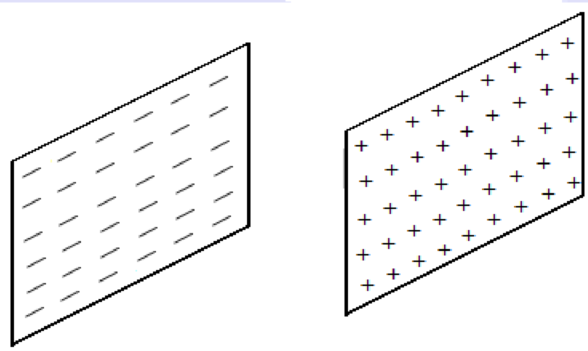


I Region	II Region	III Region
E = –E$_1$ – E$_2$ $E = -\left(\frac{\sigma}{2\,\varepsilon_0}\right) - \left(-\frac{\sigma}{2\varepsilon_0}\right)$ E = 0	E = E$_1$ – E$_2$ $E = \frac{\sigma}{2\,\varepsilon_0} - \left(\frac{-\sigma}{2\varepsilon_0}\right)$ $E = \frac{2\sigma}{2\,\varepsilon_0}$ $E = \frac{\sigma}{2\varepsilon_0}$	E = E$_1$ + E$_2$ $E = \frac{\sigma}{2\,\varepsilon_0} + -\frac{\sigma}{2\varepsilon_0}$ E = 0

Case – 4: When one thin sheet is negative and other is positive.

$E_1 = -\frac{\sigma}{2\varepsilon_0}$

$E_2 = \frac{\sigma}{2\varepsilon_0}$

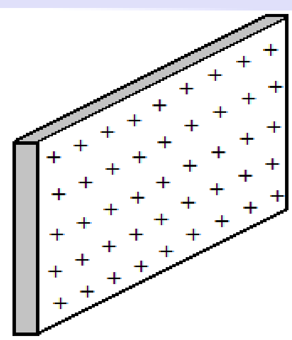


I Region	II Region	III Region
E = –E$_1$ – E$_2$ $E = -\left(-\frac{\sigma}{2\,\varepsilon_0}\right) - \left(\frac{\sigma}{2\varepsilon_0}\right)$ E = 0	E = E$_1$ – E$_2$ $E = -\frac{\sigma}{2\,\varepsilon_0} - \left(\frac{\sigma}{2\varepsilon_0}\right)$ $E = -\frac{\sigma}{\varepsilon_0}$	E = E$_1$ + E$_2$ $E = -\frac{\sigma}{2\,\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$ E = 0

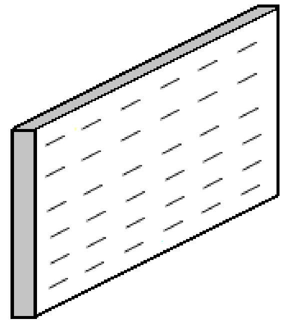
2) Thick sheet

Electric field due to positive thick sheet.

$E = \frac{\sigma}{\varepsilon_0}$



Electric field due to negative thick sheet.



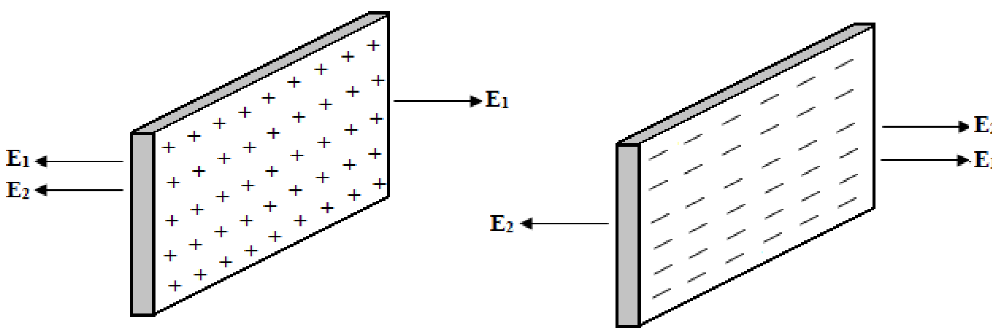
$E = -\frac{\sigma}{\varepsilon_0}$

σ is negative for negative sheet

Case – 1: Thick sheet when both are positive

$E_1 = \frac{\sigma}{\varepsilon_0}$

$E_2 = \frac{\sigma}{\varepsilon_0}$

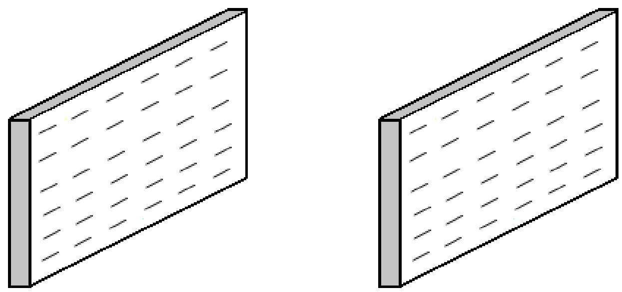


I Region	II Region	III Region
$E = -E_1 - E_2$ $E = -\frac{\sigma}{\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$ $E = -\frac{2\sigma}{\varepsilon_0}$ $E = -\frac{2\sigma}{\varepsilon_0}$	$E = E_1 - E_2$ $E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$ $E = 0$	$E = E_1 + E_2$ $E = \frac{\sigma}{\varepsilon_0} + \frac{\sigma}{\varepsilon_0}$ $E = \frac{2\sigma}{\varepsilon_0}$ $E = 2\frac{\sigma}{\varepsilon_0}$

Case – 2: Thick sheet when both are negative

$E_1 = -\frac{\sigma}{\varepsilon_0}$

$E_2 = -\frac{\sigma}{\varepsilon_0}$

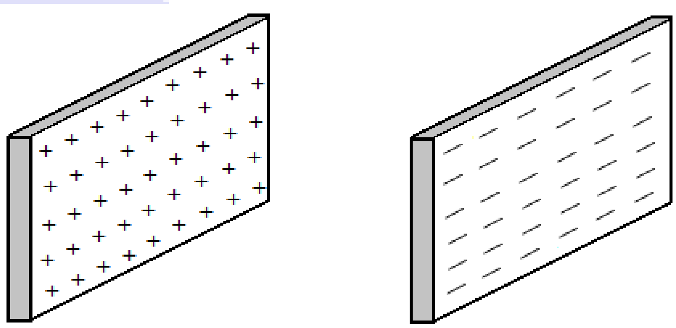


I Region	II Region	III Region
$E = -E_1 - E_2$ $E = -\left(-\frac{\sigma}{\varepsilon_0}\right) - \left(-\frac{\sigma}{\varepsilon_0}\right)$ $E = \frac{2\sigma}{\varepsilon_0}$	$E = E_1 - E_2$ $E = -\frac{\sigma}{\varepsilon_0} - \left(-\frac{\sigma}{\varepsilon_0}\right)$ $E = 0$	$E = E_1 + E_2$ $E = -\frac{\sigma}{\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$ $E = -2\frac{\sigma}{\varepsilon_0}$

Case – 3: When one thick sheet is positive and other is negative

$E_1 = \frac{\sigma}{\varepsilon_0}$

$E_2 = -\frac{\sigma}{\varepsilon_0}$

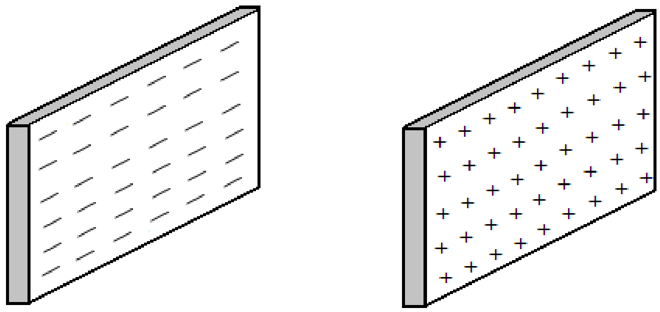


I Region	II Region	III Region
$E = -E_1 - E_2$ $E = -\left(\frac{\sigma}{\varepsilon_0}\right) - \left(-\frac{\sigma}{\varepsilon_0}\right)$ $E = 0$	$E = E_1 - E_2$ $E = \frac{\sigma}{\varepsilon_0} - \left(-\frac{\sigma}{\varepsilon_0}\right)$ $E = \frac{\sigma}{\varepsilon_0} + \frac{\sigma}{\varepsilon_0}$ $E = \frac{2\sigma}{\varepsilon_0}$	$E = E_1 + E_2$ $E = \frac{\sigma}{\varepsilon_0} + \frac{-\sigma}{\varepsilon_0}$ $E = 0$

Case – 4: When one thick sheet is negative and other is positive.

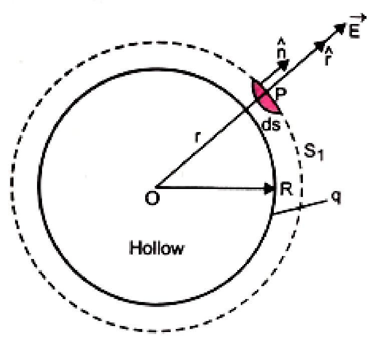
$E_1 = -\frac{\sigma}{\varepsilon_0}$

$E_2 = \frac{\sigma}{\varepsilon_0}$



I Region	II Region	III Region
$E = -E_1 - E_2$ $E = -\left(-\frac{\sigma}{\varepsilon_0}\right) - \left(\frac{\sigma}{\varepsilon_0}\right)$ $E = 0$	$E = E_1 - E_2$ $E = -\frac{\sigma}{\varepsilon_0} - \left(\frac{\sigma}{\varepsilon_0}\right)$ $E = -\frac{2\sigma}{\varepsilon_0}$	$E = E_1 + E_2$ $E = -\frac{\sigma}{\varepsilon_0} + \frac{\sigma}{\varepsilon_0}$ $E = 0$

19- Electric field density due to uniformly charge spherical shell

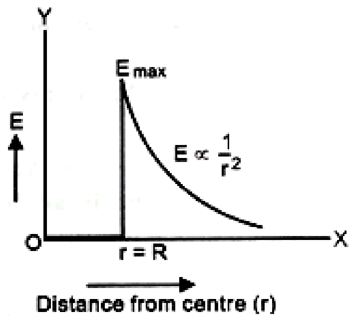


$E_{on\ surface} = \frac{k\,q}{r^2}$

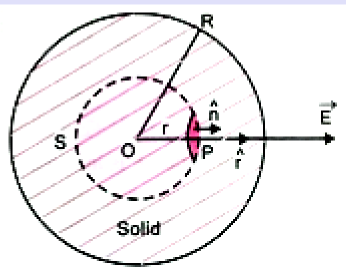
$E_{out\ surface} = \frac{k\,q}{R^2}$

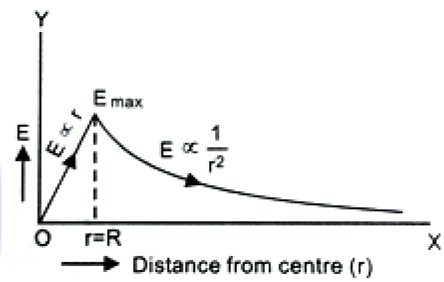
$E_{inside} = 0$ (As charge inside spherical shell is zero, the Gaussian surface encloses no charge)

Variation of electric field intensity E with distance from the centre of a uniformly charged spherical shell.



20- Electric field density due to uniformly charge solid spherical shell





Let r is the radius of the solid sphere and ρ is density of solid sphere.

Let ϵ is a electrical permittivity of solid sphere. The electric field due to the sphere,

$E = \frac{r\,\rho}{3\epsilon}$

At the centre of the sphere, r = 0, $\therefore E = 0$

At the surface of the sphere, r = R

$\therefore \quad E = \frac{R\,\rho}{3\epsilon} \Rightarrow$ maximum

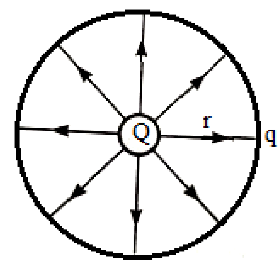
Electric Potential and Capicitor

Part – 2

1- Let q charge move in electric field of charge Q. Then q will experience force by Q.

Electric potential (V) at any point in a region of electrostatic field is minimum work done in carrying a unit positive charge from infinity to that point.

Electric Potential due to charge (Q) at distance (r)



$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

Unit of electrical potential is volt (V)

$$1V = \frac{1\,J}{1\,C} = 1\ JC^{-1} = 1\ Nm\ C^{-1}$$

Dimension of electrostatic potential = $V = \frac{W}{q} = \frac{MLT^{-2}}{AT} = ML^2T^{-3}A^{-1}$

It is a scalar quantity.

Electric potential energy (W or U) of charge q at point P in electrostatic field due to any charge configuration as the work done by external force in bringing the charge q from infinity to that point.

Electric potential energy = Electric potential by source charge (Q) × charge (q)

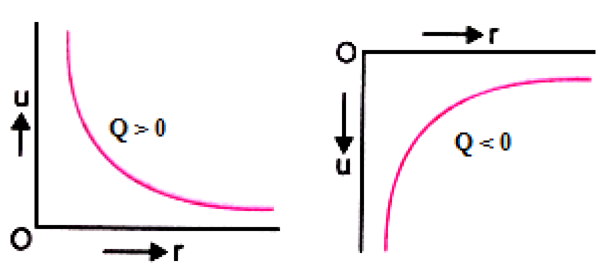
$$W = U = V.\,q = \frac{Q\,q}{4\pi\varepsilon_0 r}$$

Unit of electrical potential energy is joule (J)

Dimension of electrostatic potential energy = W = U = MLT^{-2}

It is a scalar quantity.

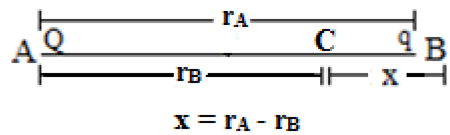
Variation of Electric Potential Energy (U) with Distance (r)



When charge (q) move from (B) to (C), charge in electric potential energy is

ΔU = Potential energy at B – Potential energy at A

$$\Delta U = U_B - U_A = (V_B - V_A)\, q = \left(\frac{kQ}{r_B} - \frac{kQ}{r_A}\right) q$$

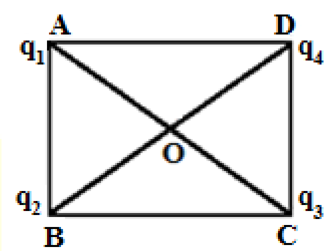


$$\frac{W_{AB}}{q_0} = V_B - V_A$$

$$\frac{W_{BA}}{q_0} = V_A - V_B$$

$$\frac{W_{AB}}{q_0} + \frac{W_{BA}}{q_0} = \frac{W_{ABA}}{q_0} = (V_B - V_A) + (V_A - V_B) = \text{Zero}$$

Potential Energy of system



Work done by (q_1) = w_1 = Potential × charge

$= V \times q_1 = 0 \times q_1 = 0$

Work done by (q_2) = w_2 = (Potential by q_1) × q_2

$$= \left[\frac{Kq_1}{r_{12}} \times q_2\right]$$

Work done by (q_3) = w_3 = (Potential by q_1 & q_2) × q_3

$$= \left[\frac{Kq_1q_3}{r_{13}}\right] + \left[\frac{Kq_2q_3}{r_{23}}\right]$$

Work done by (q_4) = w_4 = (Potential by $q_1 + q_2 + q_3$) × q_4

$= [V_1q_4] + [V_2q_4] + [V_3q_4]$

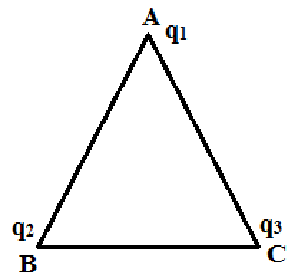
$$= \left[\frac{Kq_1q_4}{r_{14}}\right] + \left[\frac{Kq_2q_4}{r_{24}}\right] + \left[\frac{Kq_3q_4}{r_{34}}\right]$$

Total work done

$w = w_1 + w_2 + w_3 + w_4$

$$w = 0 + \frac{Kq_2q_1}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}} + \frac{Kq_1q_4}{r_{14}} + \frac{Kq_2q_4}{r_{24}} + \frac{Kq_3q_4}{r_{34}}$$

2-Potential Energy of system



Work done by (q_1) = w_1 = Potential × charge

= V × q_1 = 0 × q_1 = 0

Work done by (q_2) = w_2 = (Potential by q_1) × q_2

$$= \left[\frac{Kq_1}{r_{12}} \times q_2\right]$$

Work done by (q_3) = w_3 = (Potential by q_1 & q_2) × q_3

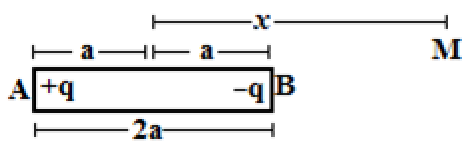
$$= \left[\frac{Kq_1q_3}{r_{13}}\right] + \left[\frac{Kq_2q_3}{r_{23}}\right]$$

Total work done

$w = w_1 + w_2 + w_3$

$$w = 0 + \frac{Kq_2q_1}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}}$$

Electric Potential at Axial Point



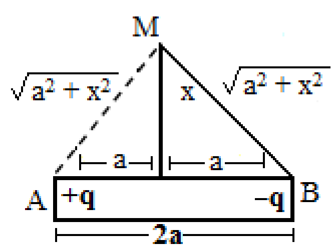
Let V_A is potential at M by charge at A

Let V_B is potential at M by charge at B

$$V = V_A + V_B = \frac{Kq}{(x+a)} + \frac{-Kq}{(x-a)}$$

$$V = \frac{-2Kqa}{(x^2-a^2)}$$

Electric Potential at Equatorial Point



Let V_A is potential at M by charge at A

Let V_B is potential at M by charge at B

$$V = V_A + V_B = \frac{Kq}{\sqrt{x^2+a^2}} - \frac{Kq}{\sqrt{x^2+a^2}}$$

V = 0

Note:

Potential and Work done are scalar quantity so direction will not consider. They will add as a numerically.

Equipotential Surface

The surface where potential is same at every point

Let (q_0) charge moves

$W_{AB} = V_{AB} \times q_0$

$W_{AB} = [V_B - V_A] \times q_0$

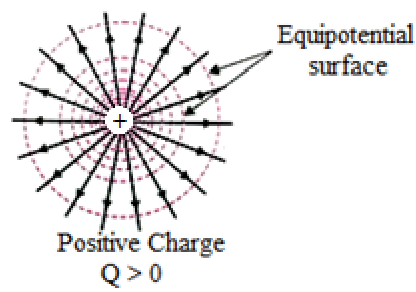
$\therefore\ V_A = V_B = V$

$W_{AB} = [V - V] \times q_0 = 0$

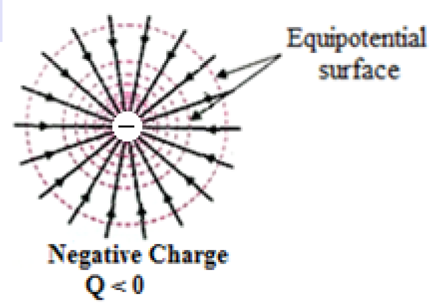
$W_{AB} = 0$ [work done on equipotential surface is zero]

Equipotential Surface

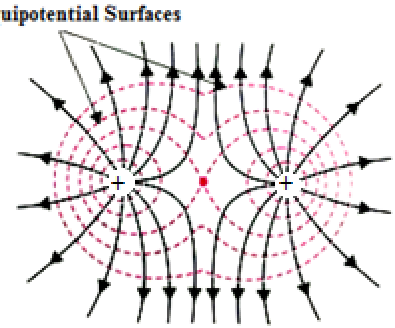
1) When charge is positive ($Q > 0$)



2) When charge is negative ($Q < 0$)

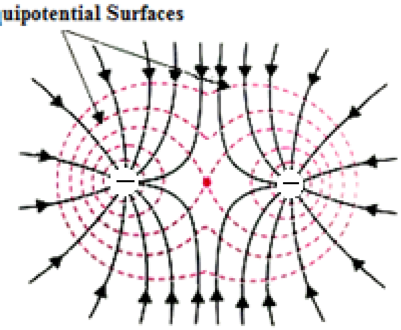


3) When both charge are positive



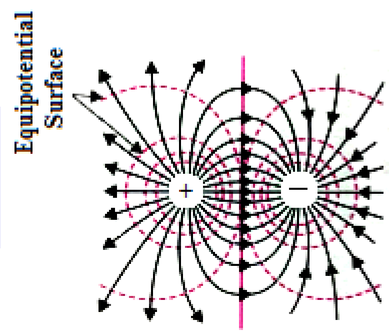
Two equal positive charges

4) When both charge are negative



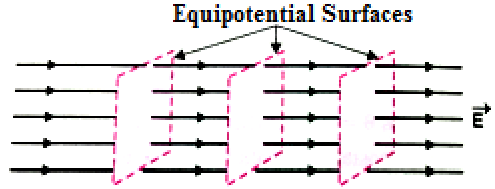
Two equal negative charges

5) Equipotential Surface of Electric Dipole



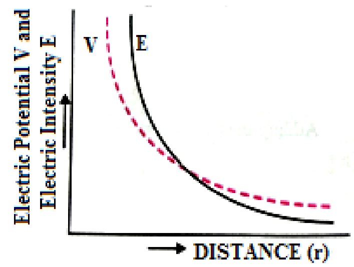
Electric Dipole

6) Equipotential Surface of Uniform Electric Field

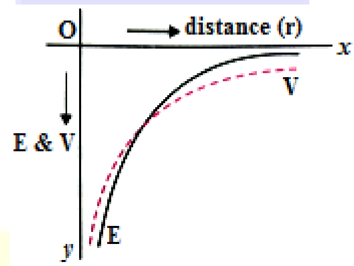


Uniform Electric Field

<u>Graph between Electrical Potential (V) and Electric Field Intensity (E) with distance (r) when charge is positive.</u>

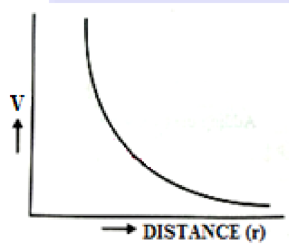


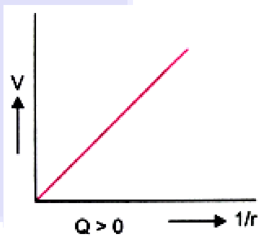
<u>Graph between Electrical Potential (V) and Electric Field Intensity (E) with distance (r) when charge is negative.</u>

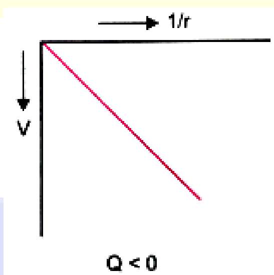


Variation of Electric Potential with Distance r

$$V = \frac{KQ}{r}$$







<u>Relation between E & V (Potential)</u>

<u>Potential Gradient</u>

Electric Field is negative of potential gradient

$$E = \frac{dV}{dr}$$ = Potential change with distance = Potential gradient

E = –Potential gradient

$$E_x = \frac{dV_x}{dx} \qquad (x - \text{axis})$$

$E_y = \frac{dV_y}{dy}$ (y – axis)

$E_z = \frac{dV_z}{dz}$ (z – axis)

Electric Potential is negative

$E = -\frac{dv}{dr}$

$\int dv = \int E.\,dr$

$V = -\int E.\,dr$

CAPACITOR

1-Electrical capacitance of a conductor is related to its ability to store the electric charge or energy.

Let charge (q) is store in capacitor (C)

q = CV

Capacitance is a unit of Capacitor

Unit of capacitance is farad.

q = CV

$C = \frac{q}{V}$

$1 \text{ farad (F)} = \frac{1 \text{ coulomb (C)}}{1 \text{ volt (V)}}$

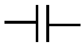
Dimension of Capacitor

q = CV [$\because W = V \times q,\ V = \frac{W}{q},\ W = F.S$]

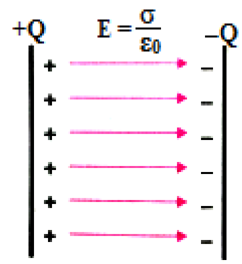
$C = \frac{q}{V}$

$C = \frac{q}{W/q} = \frac{q^2}{W}$

$C = \frac{[AT]^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}A^2T^4]$

It is denoted by 

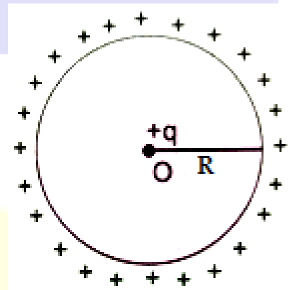
Electric Field in Capacitor



$= E = \dfrac{\sigma}{\varepsilon_0}$, $\sigma = q/A$

where σ is surface charge density

2-Capacitance of Spherical Conductor of radius (R)

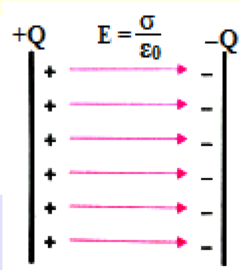


$C = 4\pi\varepsilon_0 R$

Note: Ratio of Capacitance of two spherical conductor of radius R_1 & R_2

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

3- Capacitance of Parallel Plate Capacitor



$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

4- Energy stored in Capacitor

$$E = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$$

Where C, V and q are capacitance, potential and charge respectively.

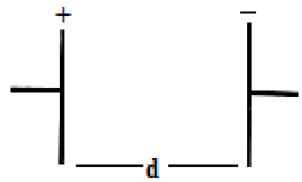
5- Energy stored per unit volume or the energy density of electric field of a capacitor

$$u = \frac{\text{total energy (U)}}{\text{volume (v)}} = \frac{\text{total energy (U)}}{\text{Area of capacitor} \times \text{Distance between the plates}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)\left(\frac{E^2 d^2}{Ad}\right)$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

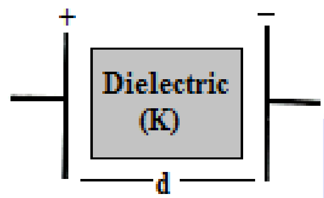
Note:

When capacitor filled by air then capacitance will be C_0



$C_0 = \frac{\epsilon_0 A}{d}$

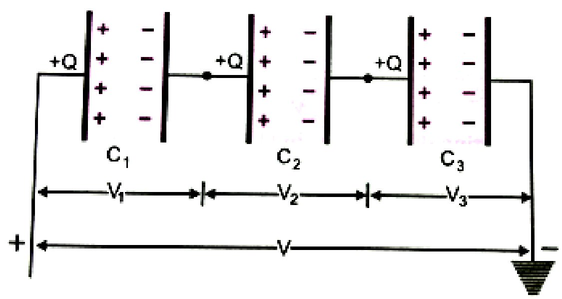
When capacitor filled by dielectric then capacitance will be C



$C = K.C_0$

$C = K\frac{\epsilon_0 A}{d}$

6- Capacitor one in series



$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\dots + \frac{1}{C_n}$

In series combination charge is same on every capacitor

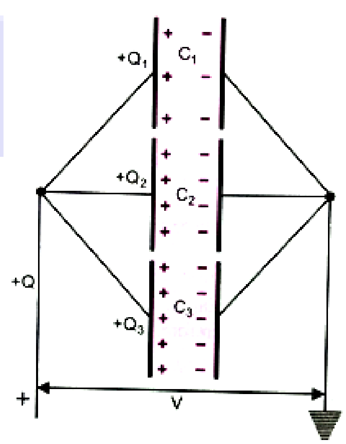
and Potential will be different for every capacitor

7- Capacitor one in parallel

$C = C_1 + C_2 + C_3 + \dots\dots + C_n$

In series combination charge is different on every capacitor

and Potential will be same for every capacitor

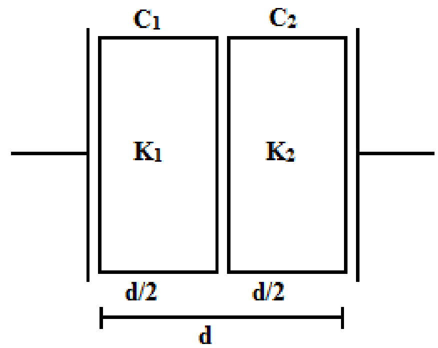


Ques. Find the capacitance of diagram.

Sol. Let C_1 and C_2 are two capacitance.

They are connected in series

Distance between the capacitor will be half for both capacitors



$$C_1 = \frac{\varepsilon_0 A K_1}{d/2}$$

$$C_2 = \frac{\varepsilon_0 A K_2}{d/2}$$

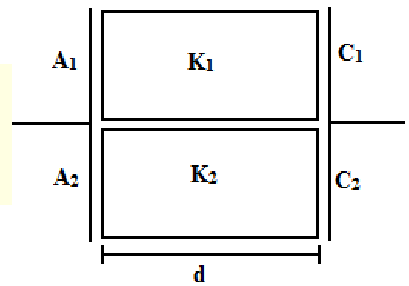
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Ques. Find the capacitance of diagram.

Sol: Let C_1 and C_2 are two capacitance.

They are connected in parallel

Distance between the capacitor will be half for both capacitors



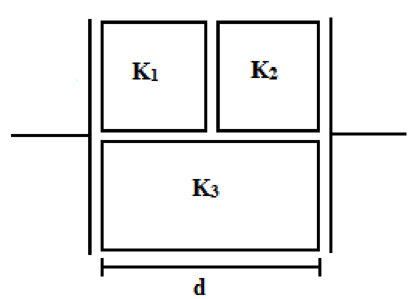
$$C_1 = \frac{\varepsilon_0 K_1}{d}\frac{A}{2}$$

$$C_2 = \frac{\varepsilon_0 K_2}{d}\frac{A}{2}$$

$C = C_1 + C_2$

Ques. Find the capacitance of diagram.

Sol. C_1 and C_2 are connected in series



$$C_1 = \frac{K_1 \times \varepsilon_0}{d}\frac{A}{2}$$

$$C_2 = \frac{K_2 \times \varepsilon_0}{d/2}\frac{A}{2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

C and C_3 are in parallel

$$C_3 = \frac{K_3 \times \varepsilon_0}{d/2}\frac{A}{2}$$

$C_T = C + C_3$

Note:

When a dielectric slab is introduced in between the plates of a charged capacitor with battery connected across the plates:

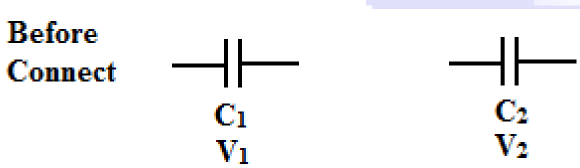
- Potential V remains constant, ($V = V_0$)
- Capacity C increases, ($C = KC_0$)
- Charge Q increases, ($Q = CV$)
- Electric field E decreases ($E = E_0/K$)
- Energy U increases, $\left(U = \frac{1}{2}CV^2 = KU_0\right)$

However, when battery across the plates of charged condenser is off and dielectric slab is introduced between its plates:

- Charge Q remains constant, ($Q = Q_0$)
- Capacity C increases, ($C = KC_0$)
- Potential V decreases, $\left(V = \frac{Q}{C} = \frac{V_0}{K}\right)$
- Electric field E decreases $\left(E = \frac{V}{d} = \frac{V_0}{Kd}\right)$
- Energy U decreases $\left(U = \frac{Q^2}{2C} = \frac{U_0}{K}\right)$

Common Potential and Loss of energy in capacitor

Case 1: Capacitor before connecting



Capacitor 1

Charge in capacitor (C_1) = $q_1 = C_1V_1$

Energy in capacitor (C_1) = $u_1 = \frac{1}{2}C_1V_1^2$

Capacitor 2

Charge in capacitor (C_2) = $q_2 = C_2V_2$

Energy in capacitor (C_2) = $u_2 = \frac{1}{2}C_2V_2^2$

Case 2: Capacitor after connecting

After Connect – Charge move from higher potential to lower potential till, potential became equal.

Capacitor 1

Charge in capacitor (C_1) = $q_1' = C_1V$

Energy in capacitor (C_1) = $u_1' = \frac{1}{2}C_1V^2$

Capacitor 2

Charge in capacitor (C_2) = $q_2' = C_2V$

Energy in capacitor (C_2) = $u_2' = \frac{1}{2}C_2V^2$

Charge always conserved

Charge before connecting = Charge after connecting

So, $q_1 + q_2 = q_1' + q_2'$

$C_1V_1 + C_2V_2 = C_1V + C_2V$

$C_1V_1 + C_2V_2 = (C_1 + C_2)V$

Common potential = $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Energy before connecting (U_B)

$U_B = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

Energy after connecting (U_A)

$U_A = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$

Energy loss = Energy before connecting – Energy after connecting

$\Delta E = U_B - U_A = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$

From:

Tarun Kumar Gautam

14-year experienced in teaching Physics for NEET-JEE

Jamia Hmadard (HSC)