BASIC PHYSICS

TOPIC 1: SOME IMPORTANT CONVERSION FACTORS

LENGTH

- 1m = 100 cm = 1000mm = 3.28 ft. = 39.37 in = 1.0936 yd (yard)
- 1km = 0.6215 mi (mile)
- 1mi = 1609 m
- 1n mi (nautical mile) = 1852 m
- 1 in = 2.54 cm
- 1 ft = 12 in = 30.48 cm
- 1 bohr radius = 0.529 Å
- 1AU (Astronomical unit) = 1.49×10^{11} m (Average distance between sun and earth)
- 1 *ly* (light year) = 9.461×10^{15} m (Distance travelled by light in vacuum in one year)
- 1 parsec or parallactic second = 3.08×10^{16} m = 3.26 *ly* (Distance at which an arc of length 1AU subtends an angle of one second at a point)

MASS

- 1kg = 1000g = 2.2 *l*b (pound)
- 1 quintal = 100 kg
- 1 ton = 907.2 kg
- 1 metric tonne = 1000kg = 10^6 g
- 1*l*b = 454 g
- 1 slug = 14.59 kg
- 1 ounce = 28.35 g
- 1 amu = 1.6606×10^{-27}kg = 931.5 MeV/c^2
- 1 Chandra Shekhar Limit = 1.4 M_{sun}

TIME

• 1h	= 60 min	= 3600 s
• 1d	= 24 h	= 1440 min = 86.4×10^3 s
• 1y	= 365.24 d	= 31.56×10^6 s
• 1 shake	= 10^{-8} s	

AREA

- $1m^2$ = $10^4\,cm^2$
- $1km^2$ = 0.386 mi^2 = 247 acres
- 1 acre = 43,560 ft^2 = $4047m^2$ = 0.4047 hectare
- 1 hectare = $10^4\,m^2$ = 2.47 acres
- 1 barn = $10^{-28}m^2$ (for measuring cross – sectional areas in sub – atomic particle collisions)

VOLUME

- $1m^3$ = 10^6cm^3 = 10^6 cc = 10^3L = 35.31 ft^3
- 1 gal (gallon) = 3.786 L (in U.S.A.) or 4.54 L (in U.K.)

DENSITY

- 1 kg/m^3 = 10^{-3} g/cm^3 = 10^{-3} kg/L

SPEED

• 1 km h^{-1}	= 5/18 m/s	or	0.2778 m/s	= 0.6215 mi/h
• 1 mi h^{-1}	= 0.4470 m/s	=	1.609 km/h	= 1.467 ft/s
• 1 m s^{-1}	= 18/5 km/h	or	3.6 km/h	= 2.24 mi/h

ACCELERATION

- g = 9.8 m/s^2 (MKS unit) = 980 cm/s^2 (CGS unit) = 32 ft/s^2 (FPS unit)

ANGLE AND ANGULAR SPEED

- π rad = 180°
- 1 rad = 180°/π or 57.30°
- 1° = 1.745×10^{-2} rad = 60' = 1/360 revolution
- 1 rev = 360° = 2π rad
- 1' (min) = 60'' (second)

- 1 rev/ min = 0.1047 rad/s ≈ 0.1 rad/s
- 1 rad/s = 9.549 rev/min

FORCE

- 1N = 10^5 dyne = 7.23 poundal
- 1 kg – wt = 1 kg – f = 9.8 N
- 1 g – wt = 1 g – f = 980 dyne
- 1 *l*b – wt = 1 *l*b – f = 32 poundal

PRESSURE

- 1Pa = $1N/m^2$ = 10 $dyne/cm^2$
- 1 bar = 10^5 Pa = 10^6 $dyne/cm^2$
- 1 atrn = 1.01325 bar = 1.01×10^5 Pa = 1.01×10^6 $dyne/cm^2$ = 760 mm of Hg column
- 1 torr = 1mm of Hg column = 153.32 Pa

WORK ENERGY

- 1J = 10^7erg = 0.239 cal
- 1eV = 1.6×10^{-19}J
- 1 amu = 931 MeV = 1.492×10^{-10} J
- 1 cal = 4.186 J
- 1 kWh = 3.6 MJ = 860 kcal
- 1 Btu (British thermal unit) = 1055J

POWER

- 1 hp (horse power) = 745.7W ≈ 746 W
- 1kW = 1000W = 1.34 hp
- 1 W (watt) = 1 J/s
- 1 cal/s = 4.186 W

TEMPERATURE

- K (kelvin) = [°C + 273°] = [°F + 459.67] / 1.8 = °R/1.8
- °F = °C × 9/5 + 32

ELECTRIC CHARGE

- 1C (coulomb) = 3×10^9 stat coulomb = 0.1 ab coulomb
- 1 esu = 1 stat coulomb = 3.33×10^{-10} coulomb
- 1 emu = 1 ab coulomb = 10 coulomb
- 1 A – h = 3600 (coulomb)

ELECTRIC CURRENT

- 1 A (ampere) = 3×10^9 stat ampere (esu of current) = 0.1 ab ampere (emu of current)

RADIOACTIVITY

- 1 Bq (bacquerel) = 1 dps (disintegration per second)
- 1 Ci (curie) = 3.7×10^{10} dps = 3.7×10^{10} Bq = 3.7×10^4 Rd
- 1 Rd (rutherford) = 10^6 dps = 10^6 Bq

OTHERS

- 1 weber = 10^8 maxwell (for Magnetic flux)
- 1 T (tesla) = 1 weber/m^2 = 10^4G (gauss) (for Magnetic flux density)
- 1 orested = 79.554 A/m (for Intensity of Magnetic field)
- 1 poiseuille (N- s/m^2 or Pa – s) = 10 poise (Dyne – s/cm^2) (for Viscosity)

TOPIC 2: NUMERICAL CONSTANTS

I. FUNDAMENTAL PHYSICAL CONSTANTS

Name	Symbol	Value	Computational Value
Speed of light	**c**	2.99792458×10^8 m/s	3.00×10^8 m/s
Elementary constant	**e**	$1.60217653 \times 10^{-19}$ C	1.60×10^{-19} C
Gravitational constant	**G**	6.6742×10^{-11} N – m^2/kg^2	6.67×10^{-11} N – m^2/kg^2
Universal gas constant	**R**	8.314472 J/mol – K	8.31 J/mol – K
Avagadro's constant	N_A	6.0221415×10^{23} molecules /mol	6.02×10^{23} molecules /mol
Boltzmann constant	**k**	$1.3806505 \times 10^{-23}$ J/K	1.38×10^{-23} J/K
Stefan – Boltzmann constant	**σ**	5.670400×10^{-8} W/m^2- K^4	5.67×10^{-8} W/m^2- K^4
Molar volume of ideal gas at STP	V_m	22.413996 litre/mol	22.4 litre/mol
Planck's constant	**h**	$6.6260693 \times 10^{-34}$ J – s	6.62×10^{-34} J – s
Mass of electron	m_e	$9.1093826 \times 10^{-31}$ kg	9.11×10^{-31} kg

Mass of proton	$\mathbf{m_p}$	$1.67262171 \times 10^{-27}$ kg	1.67×10^{-27} kg
Mass of neutron	$\mathbf{m_n}$	$1.67492728 \times 10^{-27}$ kg	1.68×10^{-27} kg
Permeability of free space	$\boldsymbol{\mu_0}$	$4\pi \times 10^{-7}$ Wb/A – m	1.27×10^{-6} Wb/A – m
Permittivity of free space	$\boldsymbol{\varepsilon_0}$	$8.85418781762 \times 10^{-12}$ $C^2/N - m^2$	8.85×10^{-12} $C^2/N - m^2$
	$\mathbf{\frac{1}{4\pi\varepsilon_0}}$	8.987551787×10^9 $N - m^2/C^2$	9.0×10^9 $N - m^2/C^2$
STP means standard temperature and pressure: 0°C and 1.0atm			
II. OTHER USEFUL PHYSICAL CONSTANTS			
Name	**Symbol**	**Value**	**Computational Value**
Mechanical equivalent of heat	J	4.186 J/cal	4.2 J/cal
Standard atmospheric pressure	1 atm	1.01325×10^5 Pa	1.103×10^6 Pa
Absolute zero	0 K	–273.15° C	–273° C
Electron volt	1 eV	$1.60217653 \times 10^{-19}$ J	1.60×10^{-19} J
Atomic mass unit	1 u	$1.66053886 \times 10^{-27}$ kg	1.66×10^{-27} kg
Electron rest energy	m_ec^2	0.510998918 MeV	0.511 MeV
Ratio of proton mass to electron mass	$\frac{m_p}{m_e}$	1836.1526675	1840
Electron charge to mass ratio	$\frac{e}{m_e}$	$1.758820174 \times 10^{11}$ C/kg	1.76×10^{11} C/kg
Bohr magneton	μ_B	$9.27400899 \times 10^{-24}$ J/T	9.2×10^{-24} J/T
Bohr radius	a_0	$5.291772083 \times 10^{-11}$m	5.29×10^{-11}m
Rydberg constant	R_H	1.097373156×10^7 m^{-1}	1.10×10^7 m^{-1}
Energy equivalent of 1u	mc^2	931.49404 MeV	931.5 MeV
Acceleration due to gravity (standard)	g	$9.80665 m/s^2$	$9.81 m/s^2$

TOPIC 3: SOME IMPORTANT NOTATIONS

Megawat	$1MW = 10^6W$
Centimeter	$1cm = 10^{-2}m$
Kilometre	$1km = 10^3m$
Millivolt	$1mV = 10^{-3}V$
Kilowatt – hour	$1kWh = 10^3Wh = 3.6 MJ = 3.6 \times 10^6J$
Microampere	$1\mu A = 10^{-6}A$
Angstrom	$1A = 0.1nm = 10^{-10}m$
Nanosecond	$1rs = 10^{-9}s$
Picofarad	$1pF = 10^{-12}F$
Microsecond	$1\mu s = 10^{-6}s$
Gigahertz	$1GHz = 10^9$ Hz
Micron	$1\mu m = 10^{-6}m$
$1ms^{-1}$	1 metre per second
1ms	1 millisecond
1Cm	1 coulomb metre
1cm	1 centimetre
$10^{-9}m$	1 nm (nanometer)

10^{-6}m	1 μm (micron)
10^{-12}F	1 pF (picofarad)
10^{9}W	1 GW (giga watt)
cm^3	$(cm)^3 = (0.01m)^3 = (10^{-2}m)^3 = 10^{-6}m^3$
mA^2	$(mA)^2 = (0.001A)^2 = (10^{-3}A)^2 = 10^{-6}A^2$

SOME IMPORTANT UNIT

Physical Quantity	Units
Force (F)	Newton (N)
Energy (E)	Joule (J)
Electric current (I)	Ampere (A)
Temperature (T)	Kelvin (K)
Frequency (ν)	Hertz (Hz)
Length (L)	Metre (m)
Mass (M)	Kilogram (kg)
Luminous Intensity (cd)	Candela (cd)
Time (t)	Second (s)
Quantity of matter (mol)	Mole
Plane Angle	Radian (rad)
Solid Angle	Steradian (sr)

PREFIXES USED FOR DIFFERENT POWERS OF 10

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

BASIC MATHEMATICS

TOPIC 1: TRIGONOMETRY

ANGLE

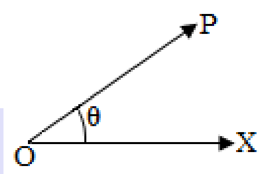
Consider a revolving line OP.

Suppose that is revolves in anticlockwise direction starting from its initial position OX. The angle is defined as the amount of revolution that the revolving line makes with its initial position.

From fig. the angle covered by the revolving line OP is $\theta = \angle POX$ The angle

is taken **positive** if it is traced by the revolving line in anticlockwise direction and

is taken **negative** if it is covered in clockwise direction.

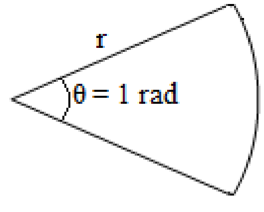


$1° = 60'$ (minute)

$1' = 60''$ (second)

1 right angle = 90° (degrees) also 1 right angle = $\frac{\pi}{2}$ rad (radian)

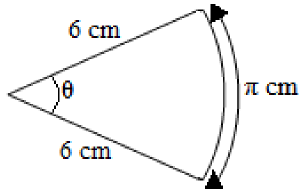
One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle. 1 rad = $\frac{180°}{\pi} \approx 57.3°$



To convert an angle from degree to radian multiply it by $\frac{\pi}{180°}$

To convert an angle from radian to degree multiply it by $\frac{180°}{\pi}$

Example 1: A circular arc is of length π cm. Find angle subtended by it at the centre in radian and degree.



Solution:

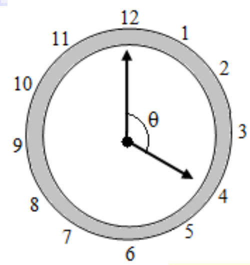
$\theta = \frac{s}{r} = \frac{\pi\ cm}{6\ cm} = \frac{\pi}{6}$ rad = 30° As 1 rad = $\frac{180°}{\pi}$ So $\theta = \frac{\pi}{6} \times \frac{180°}{\pi} = 30°$

Example 2: When a clock shows 4 o clock, how much angle do its minute and hour needles make?

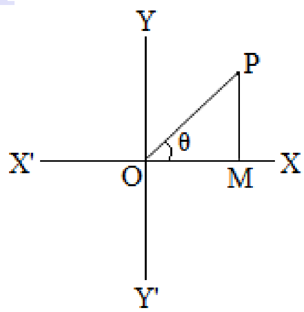
(1) 120° (2) $\frac{\pi}{3}$ rad (3) $\frac{2\pi}{3}$ rad (4) 160°

Solution:

From diagram angle θ = 4 × 30° = 120° = $\frac{2\pi}{3}$ rad



TRIGONOMETRICAL RATIOS (OR T RATIOS)



$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP}$

$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$

$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$

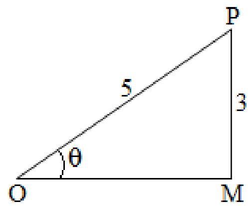
$\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM}$

$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$

Example 1: Given sin θ = 3/5. Find all the other T – ratios, if θ lies in the first quadrant.

Solution:



In Δ OMP, sin θ = $\frac{3}{5}$ so MP = 3 and OP = 5

∵ OM = $\sqrt{(5)^2 - (3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now, $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\tan\theta = \frac{MP}{OM} = \frac{3}{4}$

$\cot\theta = \frac{OM}{MP} = \frac{4}{3}$ $\sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\text{cosec}\,\theta = \frac{OP}{MP} = \frac{5}{3}$

Table: The T – ratios of a few standard angles ranging from 0° to 180°

Angle (θ)	**0°**	**30°**	**45°**	**60°**	**90°**	**120°**	**135°**	**150°**	**180°**
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ Not defined	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

TRIGONOMETRICAL RATIOS OF GENERAL ANGLES (REDUCTION FORMULAE)

(i) Trigonometric function of an angle (2nπ + θ) where n = 0, 1, 2, 3,…. will be remain same.

sin (2nπ + θ) = sin θ   cos (2nπ + θ) = cos θ   tan (2nπ + θ) = tan θ

(ii) Trigonometric function of an angle $\left(\frac{n\pi}{2} + \theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant.

sin (π – θ) = + sin θ   cos (π – θ) = – cos θ   tan (π – θ) = – tan θ

sin (π + θ) = – sin θ   cos (π + θ) = – cos θ   tan (π + θ) = + tan θ

sin (2π – θ) = – sin θ   cos (2π – θ) = + cos θ   tan (2π – θ) = – tan θ

(iii) Trigonometric function of an angle $\left(\frac{n\pi}{2} + \theta\right)$ will be changed into co – function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$\sin\left(\frac{\pi}{2} + \theta\right) = +\cos\theta$ $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$ $\tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos\theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = +\sin\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = +\cot\theta$

(iv) Trigonometric function of an angle –θ (negative angles)

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = +\cos\theta$ $\tan(-\theta) = -\tan\theta$

$\sin(90° + \theta) = \cos\theta$ $\cos(90° + \theta) = -\sin\theta$ $\tan(90° + \theta) = -\cot\theta$	$\sin(180° - \theta) = \sin\theta$ $\cos(180° - \theta) = -\cos\theta$ $\tan(180° - \theta) = -\tan\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$	$\sin(90° - \theta) = \cos\theta$ $\cos(90° - \theta) = \sin\theta$ $\tan(90° - \theta) = \cot\theta$
$\sin(180° + \theta) = -\sin\theta$ $\cos(180° + \theta) = -\cos\theta$ $\tan(180° + \theta) = \tan\theta$	$\sin(270° - \theta) = -\cos\theta$ $\cos(270° - \theta) = -\sin\theta$ $\tan(270° - \theta) = \cot\theta$	$\sin(270° + \theta) = -\cos\theta$ $\cos(270° + \theta) = \sin\theta$ $\tan(270° + \theta) = -\cot\theta$	$\sin(360° - \theta) = -\sin\theta$ $\cos(360° - \theta) = \cos\theta$ $\tan(360° - \theta) = -\tan\theta$

Example 1: Find the value of

(i) cos (–60°) (ii) tan 210° (iii) sin 300° (iv) cos 120°

Solution:

(i) $\cos(-60°) = \cos 60° = \frac{1}{2}$

(ii) $\tan 210° = \tan(180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

A FEW IMPORTANT TRIGONOMETRIC FORMULAE

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

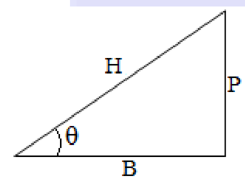
$1 + \cos A = 2\cos^2\frac{A}{2}$, $1 - \cos A = 2\sin^2\frac{A}{2}$

RANGE OF TRIGONOMETRIC FUNCTIONS

As $\sin\theta = \frac{P}{H}$ and $P \le H$ so $-1 \le \sin\theta \le 1$

As $\cos\theta = \frac{B}{H}$ and $B \le H$ so $-1 \le \cos\theta \le 1$

As $\tan\theta = \frac{P}{B}$ so $-\infty \le \tan\theta \le \infty$



Remember: $-\sqrt{a^2 + b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2 + b^2}$

TOPIC 2: COORDINATE GEOMETRY

DISTANCE FORMULA

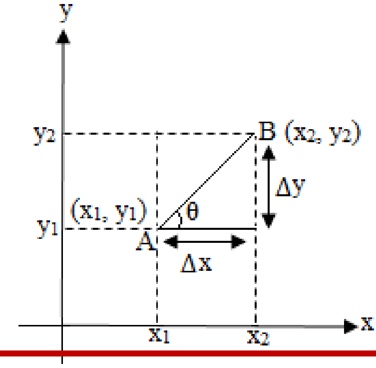
The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note: In speed $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

SLOPE OF A LINE

The slope of a line joining two points A (x_1, y_1) and B (x_2, y_2) is denoted by m and is given by

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$ [If both axes have identical scales]



Here θ is the angle made by line with positive x – axis.

Slope of a line is quantitative measure of inclination.

Example 1: For point (2, 14) find abscissa and ordinate. Also find distance from y and x axis.

Solution:

Abscissa = x – coordinate = 2 = distance from y – axis

Ordinate = y – coordinate = 14 = distance from x – axis

Example 2: Find value of a if distance between the points (–9 cm, a cm) and (3cm, 3cm) is 13cm.

Solution:

By using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow 13 = \sqrt{[3 - (-9)]^2 + [3 - a]^2}$

$\Rightarrow 13^2 = 12^2 + (3 - a)^2 \Rightarrow (3 - a)^2 = 13^2 - 12^2 = 5^2 \Rightarrow (3 - a) = \pm 5 \Rightarrow a = -2$cm or 8cm

TOPIC 3: DIFFERENTIATION

PHYSICAL MEANING OF $\frac{dy}{dx}$

(i) The ratio of small change in the function y and the variable x is called the average rate of change of y w.r.t. x.

For example, if a body covers a small distance Δs in small time Δt, then

average velocity of the body, $v_{av} = \frac{\Delta s}{\Delta t}$

Also, if the velocity of a body changes by a small amount Δv in small time Δt, then average acceleration of the body, $a_{av} = \frac{\Delta v}{\Delta t}$

(ii) When $\Delta x \rightarrow 0$ The limiting value of $\frac{\Delta y}{\Delta x}$ is $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

It is called the instantaneous rate of change of y w.r.t. x.

The differentiation of a function w.r.t. a variable implies the instantaneous rate of change of the function w.r.t. that variable.

Like wise, instantaneous velocity of the body $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

and instantaneous acceleration of the body $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

THEOREM OF DIFFERENTIATION

1. If c = constant, $\frac{d}{dx}(c) = 0$

2. y = c u, where c is a constant and u is a function of x, $\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$

3. y = u ± v ± w, where, u, v and w are functions of x, $\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$

4. y = u v where u and v are functions of x, $\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

5. $y = \frac{u}{v}$, where u and v are functions of x, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

6. $y = x^n$, n real number, $\frac{dy}{dx} = \frac{d}{dx}(x^n) = n\,x^{n-1}$

Example 1: Find $\frac{dy}{dx}$ when (i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$ (iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Solution:

(i) $y = \sqrt{x} \Rightarrow$ $\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(ii) $y = x^5 + x^4 + 7 \Rightarrow$ $\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4) + \frac{d}{dx}(7)$

$= 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2} \Rightarrow$ $\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2}) = \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) + \frac{d}{dx}(3x^{-2})$

$\frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2}) = 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)\,x^{-3}$

$2x - 2x^{-3/2} + 6x^{-3}$

FORMULAE FOR DIFFERENTIAL COEFFICIENTS OF TRIGONOMETRIC, LOGARITHMIC AND EXPONENTIAL FUNCTIONS

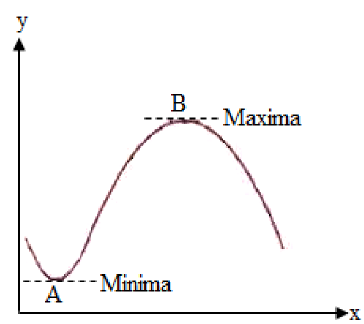
$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$ $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$ $\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(e^{ax}) = ae^{ax}$

MAXIMUM AND MINIMUM VALUE OF FUNCTION

Higher order derivatives are used to find the maximum and minimum values of a function. At the points of maxima and minima, first derivative (i.e. $\frac{dy}{dx}$) becomes zero.



At point 'A' (minima):

As we see in figure, in the neighbourhood of A, slope increases so $\frac{d^2y}{dx^2} > 0$.

Condition for minima: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

At point 'B' (maxima): As we see in figure, in the neighbourhood of B, slope decreases so $\frac{d^2y}{dx^2} < 0$.

Condition for maxima: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

Example 1: The minimum value of $y = 5x^2 - 2x + 1$ is

(1) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

Solution:

For maximum/ minimum value $\frac{dy}{dx} = 0 \Rightarrow 5\,(2x) - 2(1) + 0 = 0 \Rightarrow x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 10$ which is positive

so y has minimum value at $x = \frac{1}{5}$. Therefore $y_{min} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1 = \frac{4}{5}$

TOPIC 4: INTEGRATION

In integral calculus, the differential coefficient of a function is given. We are required to find the function. Integration is basically used for summation. $\sum$ is used for summation of discrete values, while $\int$sign is used for continous function.

If I is integration of f(x) with respect to x then $I = \int f(x)\,dx$ [we can check $\frac{dI}{dx} = f(x)$] $\because \int f'(x)\,dx = f(x) + c$

where c = an arbitrary constant

Let us proceed to obtain integral of x^n w.r.t. x. $\frac{d}{dx}(x^{n+1}) = (n + 1)\,x^n$

Since the process of integration is the reverse process of differentiation ,

$\int (n + 1)\,x^n\,dx = x^{n+1}$ or $(n + 1)\int x^n\,dx = x^{n+1} \Rightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1}$

The above formula holds for all values of n, except n = –1.

It is because, for n = –1, $\int x^n\,dx = \int x^{-1}\,dx = \int \frac{1}{x}\,dx$

$\because$ $\frac{d}{dx}(\log_e x) = \frac{1}{x}$ $\therefore \int \frac{1}{x}\,dx = \log_e x$

Similarly, the formulae for integration of some other functions can be obtained if we know the differential coefficients of various functions.

FEW BASIC FORMULAE OF INTEGRATION

Following are a few basic formulae of integration:

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$, Provided $n \neq -1$

2. $\int \sin x\, dx = -\cos x + c$ $\quad$ $(\because \frac{d}{dx}(\cos x) = -\sin x)$

3. $\int \cos x\, dx = \sin x + c$ $\quad$ $(\because \frac{d}{dx}(\sin x) = \cos x)$

4. $\int \frac{1}{x}\, dx = \log_e x + c$ $\quad$ $(\because \frac{d}{dx}(e^x) = e^x)$

Example 1: Integrate w.r.t. x. : (i) $x^{11/2}$ (ii) x^{-7} (iii) $x^{p/q}$ $(p/q \neq -1)$

Solution:

(i) $\int x^{11/2}\, dx = \frac{x^{11/2+1}}{\frac{11}{2}+1} + c = \frac{2}{13} x^{13/2} + c$

(ii) $\int x^{-7}\, dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6} x^{-6} + c$

(iii) $\int x^{p/q}\, dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q} x^{(p+q)/q} + c$

Example 2: Evaluate $\int \left(x^2 - \cos x + \frac{1}{x}\right)$

Solution:

$I = \int x^2\, dx = -\int \cos x\, dx + \int \frac{1}{x}\, dx = \frac{x^{2+1}}{2+1} - \sin x + \log_e x + c = \frac{x^3}{3} - \sin x + \log_e x + c$

DEFINITE INTEGRALS

When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

If $\frac{d}{dx}(f(x)) = f'(x)$, then

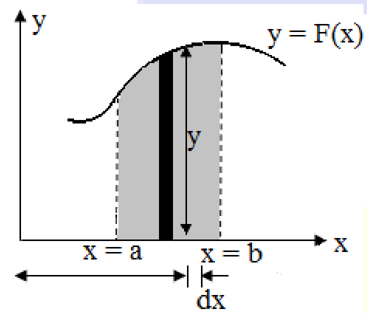
$\int f'(x)\, dx$ is called indefinite integral and $\int_a^b f'(x)\, dx$ is called definite integral

Here, a and b are called lower and upper limits of variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below:

$\int_a^b f'(x)\,dx = |f(x)|_a^b = f(b) - f(a)$

AREA UNDER A CURVE AND DEFINITE INTEGRATION



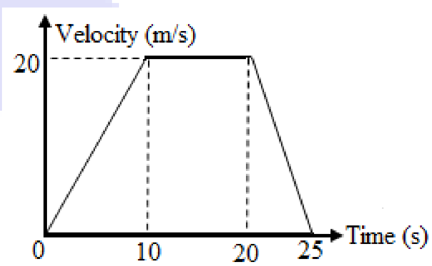
Area of small shown darkly shaded element = ydx = f(x) dx.

If we sum up all areas between x = a and x = b then

$\int_a^b f(x)dx$ = shaded area between curve and x – axis.

Example 1: The velocity – time graph of a car moving along a straight road is shown in figure. The average velocity of the car in first 25 seconds is

(1) 20m/s (2) 14m/s (3) 10m/s (4) 17.5m/s



Solution:

$$\text{Average velocity} = \frac{\int_0^{25} v\,dt}{25-0} = \frac{\text{Area of v–t graph between t = 0 to t =25s}}{25}$$

$$= \frac{1}{25}\left[\left(\frac{25+10}{2}\right)(20)\right] = 14\text{m/s}$$

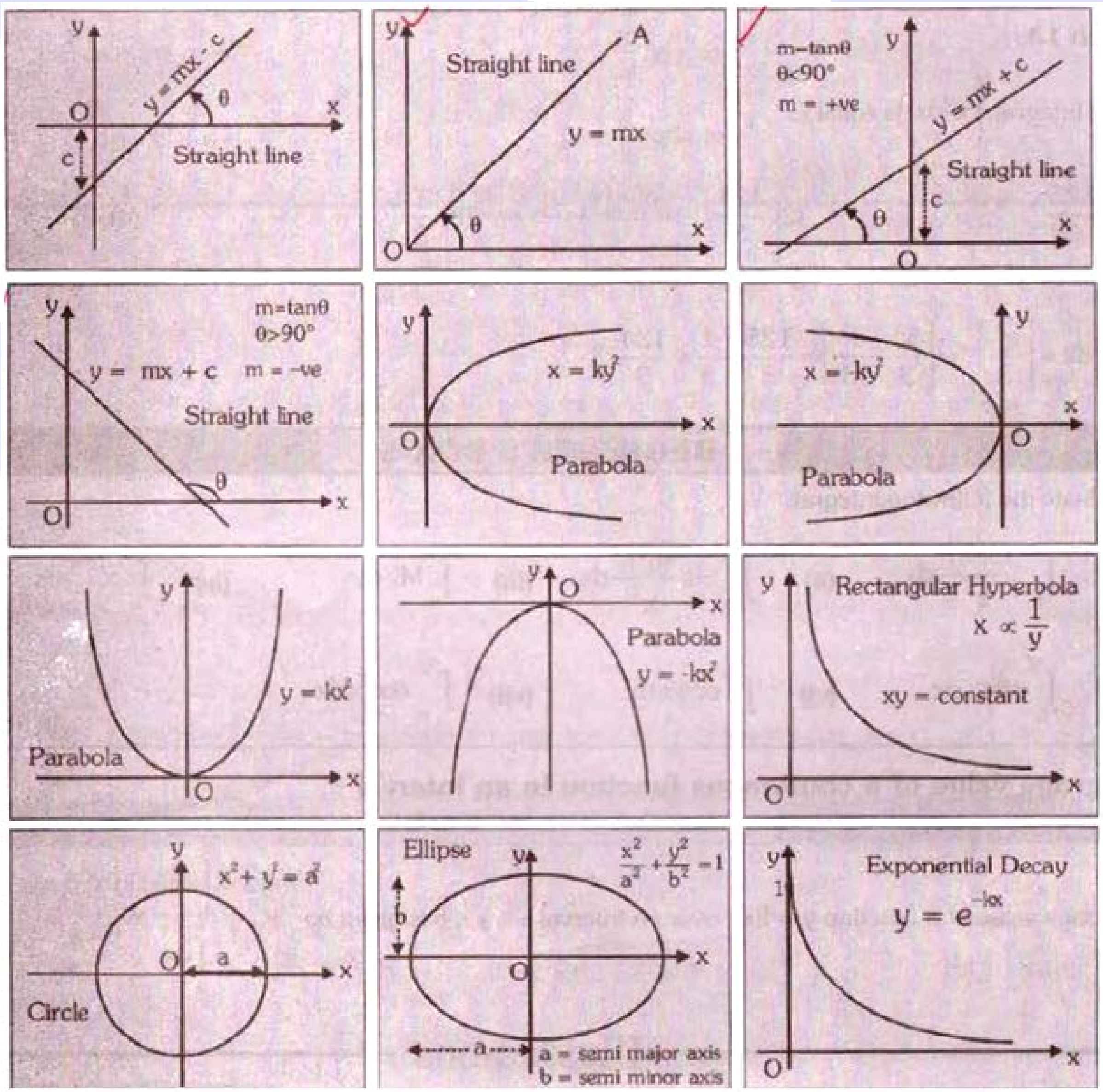
Example 2: Determine the average value of y = 2x + 3 in the interval 0 ≤ x ≤ 1.

(1) 1 (2) 5 (3) 3 (4) 4

Solution:

$$y_{av} = \frac{\int_0^1 y\,dx}{1-0} = \int_0^1 (2x+3)dx = \left[2\left(\frac{x^2}{2}\right) + 3x\right]_0^1 = 1^2 + 3(1) - 0^2 - 3(0) = 1 + 3 = 4$$

TOPIC 5: SOME STANDARD GRAPHS AND THEIR EQUATIONS



TOPIC 6: ALGEBRA

QUADRATIC EQUATION AND ITS SOLUTION

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation.

Equation $ax^2 + bx + c = 0$ is the general quadratic equation.

The general solution of the above quadratic equation of value of variable x is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots = $x_1 + x_2 = \frac{-b}{a}$ and product of roots = $x_1 x_2 = \frac{c}{a}$

For real roots discriminant $b^2 - 4ac \geq 0$ and for imaginary roots $b^2 - 4ac < 0$

Example 1: Solve the equation $2x^2 + 5x - 12 = 0$

Solution:

By comparison with the standard quadratic equation $a = 2$, $b = 5$ and $c = -12$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} = \frac{+6}{4}, \frac{-16}{4} \quad \text{or} \quad x = \frac{3}{2}, -4$$

BINOMIAL EXPRESSION

An algebraic expression containing two terms is called a binomial expression.

For example $(a + b)$, $(a + b)^3$, $(2x - 3y)^{-1}$, $\left(x + \frac{1}{y}\right)$ etc. are binomial expressions.

Binomial Theorem

$(a + b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2 \times 1} a^{n-2}b^2 + \ldots\ldots.,$ $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2 \times 1} x^2 + \ldots..$

BINOMIAL APPROXIMATION

If x is very small, compared to 1, then terms containing higher powers of x can be neglected

so $(1 + x)^n \approx 1 + nx$

For example: $(1 + x)^{3/2} = \left(1 + \frac{3}{2}x\right)$

TOPIC 7: GEOMETRY

FORMULAE FOR DETERMINATION OF AREA

1. Area of square = $(\text{side})^2$

2. Area of rectangle = length × breadth

3. Area of a triangle = $\frac{1}{2}$ (base × height)

4. Area of trapezoid = $\frac{1}{2}$ (distance between parallel sides) × (sum of parallel sides)

5. Area enclosed by a circle = πr^2 (r = radius)

6. Surface area of a sphere = $4\pi r^2$ (r = radius)

7. Area of a parallelogram = base × height

8. Area of curved surface of cylinder = $2\pi l$ (r = radius and l = length)

9. Area of ellipse = π ab (a and b are semi major and semi minor axes respectively)

10. Surface area of a cube = $6\ (\text{side})^2$

11. Total surface area of cone = $\pi r^2 + \pi r l$ where $\pi r l = \pi r\sqrt{r^2 + h^2}$ = lateral area

FORMULAE FOR DETERMINATION OF VOLUME

1. Volume of a rectangle slab = length × breadth × height = abt

2. Volume of a cube = $(\text{side})^3$

3. Volume of a sphere = $\frac{4}{3}\pi r^3$ (r = radius)

4. Volume of a cylinder = $\pi r^2 l$ (r = radius and *l* is length)

5. Volume of a cone = $\frac{1}{3}\pi r^2 h$ (r = radius and h is height)

Note: $\pi = \frac{22}{7} = 3.14$; $\pi^2 = 9.8776 \approx 10$ and $\frac{1}{\pi} = 0.3182 \approx 0.3$